Technical Appendix:
Zeros and the Gains from Openness

Timothy Uy

1 Summary

There are three sections to this appendix. Appendix A contains the proofs for all the propositions in the paper. Appendix B details the computational algorithm used to compute the approximate equilibrium. Finally, Appendix C contains an example illustrating the nonexistence of exact equilibrium and computation of approximate equilibrium in a stylized, two-country environment.

2 Appendix A: Proofs

Lemma 1. Given $f_{ij}^s = \infty$. for all (i, j) such that $e_{ij}^s = 0,\ s \in \{T, M\}$, there is no aggregate entry following policy reform.

Proof. Entry following reform requires that there exists (i, j) such that $e_{ij}^s = 0$ and $e_{ij}^{s'} = 1$ for $s \in \{T, M\}$. But this means that $\pi_{ij}^s < f_{ij}^s$ and $\pi_{ij}^{s'} \geq f_{ij}^s$. This cannot be true given that $f_{ij}^s = \infty$ and $\pi_{ij}^{s'} < \infty,\ \pi_{ij}^s < \infty$.

Lemma 2. Given $f_{ij}^s = 0$ for all (i, j) such that $e_{ij}^s = 1,\ s \in \{T, M\}$, then there is no aggregate exit post-reform.

Proof. Exit following reform requires that there exists (i, j) such that $e_{ij}^s = 1$ and $e_{ij}^{s'} = 0$ for $s \in \{T, M\}$. But this means that $\pi_{ij}^s \geq f_{ij}^s$ and $\pi_{ij}^{s'} < f_{ij}^s$. This cannot be true given that $f_{ij}^s = 0$ and $\pi_{ij}^{s'} > 0,\ \pi_{ij}^s > 0$.

Lemma 3. If there is no aggregate entry and exit following policy reform, dividend per share stays unchanged ($\pi' = \pi$).

Proof. Recall the expression for dividend per share

$$\pi = \frac{\sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j [\mu \frac{1}{\sigma} \left(\frac{p_{ij}^S}{P_i^S}\right)^{1-\sigma} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi\right) - f_{ij}^S] e_{ij}^S}{2 \sum_{i=1}^N w_i L_i}$$

Rearranging to isolate the terms that are functions of π to yield

$$
\begin{aligned}
&2\left\{\sum_{i=1}^{N} w_i L_i - \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma}\left(\frac{p_{ij}^S}{P_i^S}\right)^{1-\sigma} w_i L_i\right] e_{ij}^S\right\}\pi \\
&= \frac{1}{1-\alpha}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma}\left(\frac{p_{ij}^S}{P_i^S}\right)^{1-\sigma} w_i L_i\right] e_{ij}^S - \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{s=M,T} w_j L_j f_{ij}^S e_{ij}^S
\end{aligned}
$$

Similarly, post-reform we have

$$
\begin{aligned}
&2\left\{\sum_{i=1}^{N} w_i L_i - \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma}\left(\frac{p_{ij}^{S'}}{P_i^{S'}}\right)^{1-\sigma} w_i L_i\right] e_{ij}^{S'}\right\}\pi' \\
&= \frac{1}{1-\alpha}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma}\left(\frac{p_{ij}^{S'}}{P_i^{S'}}\right)^{1-\sigma} w_i L_i\right] e_{ij}^{S'} - \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{s=M,T} w_j L_j f_{ij}^S e_{ij}^{S'}
\end{aligned}
$$

To see that $\pi = \pi'$, it suffices to note that for any $S \in \{T, M\}, i = 1, 2, \ldots, N$ we have

$$
\begin{aligned}
\sum_{j=1}^{N} w_j L_j \left[{p_{ij}^{S'}}^{1-\sigma} e_{ij}^{S'}\right] &= {P_i^{S'}}^{1-\sigma} \\
\sum_{j=1}^{N} w_j L_j \left[{p_{ij}^{S}}^{1-\sigma} e_{ij}^{S}\right] &= {P_i^{S}}^{1-\sigma}
\end{aligned}
$$

and because there is no aggregate entry or exit, $e_{ij}^{S'} = e_{ij}^S,\ \forall (S, i, j)$, so that

$$
\sum_{s=M,T} w_j L_j f_{ij}^S e_{ij}^{S'} = \sum_{s=M,T} w_j L_j f_{ij}^S e_{ij}^S
$$

Hence

$$
\begin{aligned}
\pi &= \frac{\frac{1}{1-\alpha}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma}\left(\frac{p_{ij}^S}{P_i^S}\right)^{1-\sigma} w_i L_i\right] e_{ij}^S - \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{s=M,T} w_j L_j f_{ij}^S e_{ij}^S}{2\left\{\sum_{i=1}^{N} w_i L_i - \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma}\left(\frac{p_{ij}^S}{P_i^S}\right)^{1-\sigma} w_i L_i\right] e_{ij}^S\right\}} \\
&= \frac{\frac{1}{1-\alpha}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma}\left(\frac{p_{ij}^{S'}}{P_i^{S'}}\right)^{1-\sigma} w_i L_i\right] e_{ij}^{S'} - \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{s=M,T} w_j L_j f_{ij}^S e_{ij}^{S'}}{2\left\{\sum_{i=1}^{N} w_i L_i - \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma}\left(\frac{p_{ij}^{S'}}{P_i^{S'}}\right)^{1-\sigma} w_i L_i\right] e_{ij}^{S'}\right\}} = \pi'
\end{aligned}
$$

Proposition 1. The welfare gains computed in the limiting parameterization given by $f_{ij}^s = 0$ for all (i, j) such that $e_{ij}^s = 1,\ s \in \{T, M\}$ and $f_{ij}^s = \infty$ for all (i, j) such that $e_{ij}^s = 0,\ s \in \{T, M\}$ coincide with the welfare gains in an alternative parameterization of fixed costs where $f_{ij}^s = \underline{f}$ for all (i, j) such that $e_{ij}^s = 1,\ s \in \{T, M\}$ and $f_{ij}^s = \bar{f}$ for all (i, j) such that $e_{ij}^s = 0,\ s \in \{T, M\}$ where $\underline{f}$ is sufficiently small and $\bar{f}$ is sufficiently large to ensure that there is no aggregate entry or exit post-reform.

Proof. Denote the equilibrium objects that arise from the alternative parameterization with hats, e.g. $\hat{P}_i$, $\hat{W}$. We know given that there is no aggregate entry or exit post-reform, Lemma 3 implies that $\hat{\pi}' = \hat{\pi}$ (even as $\hat{\pi} \neq \pi$). Next note that because $\hat{e}'^{S}_{ij} = e'^{S}_{ij} = e^{S}_{ij}$ for all $i, j = 1, 2, \ldots, N$ and $S \in \{T, M\}$ we have $P_i^{S'} = \hat{P}_i^{S'}$ for all $i = 1, 2, \ldots, N$ and $S \in \{T, M\}$. The normalization stipulates that $P_0 = P_0' = 1$. Then given that welfare is given by

$$W_i = \Gamma + \log\left\{w_i L_i \left(\frac{1}{1-\alpha} + 2\pi\right)\right\} - (1-2\mu)\log P_0 - \mu \log P_i^M - \mu \log P_i^T = \Gamma \log\left(\frac{w_i L_i \left(\frac{1}{1-\alpha} + 2\pi\right)}{P_0^{1-2\mu} P_i^{M\mu} P_i^{T\mu}}\right)$$

We have that the gains from openness are given by

$$\frac{W_i'}{W_i} = \log \frac{\left(\frac{1}{1-\alpha} + 2\pi'\right) P_i^{M\mu} P_i^{T\mu}}{\left(\frac{1}{1-\alpha} + 2\pi\right) P_i^{M'\mu} P_i^{T'\mu}}$$

Hence

$$\frac{\hat{W}_i'}{\hat{W}_i} = \log \frac{\left(\frac{1}{1-\alpha} + 2\hat{\pi}'\right) \hat{P_i^{M}}^{\mu} \hat{P_i^{T}}^{\mu}}{\left(\frac{1}{1-\alpha} + 2\hat{\pi}\right) \hat{P_i^{M'}}^{\mu} \hat{P_i^{T'}}^{\mu}} = \log \frac{\hat{P_i^{M}}^{\mu} \hat{P_i^{T}}^{\mu}}{\hat{P_i^{M'}}^{\mu} \hat{P_i^{T'}}^{\mu}} = \log \frac{P_i^{M\mu} P_i^{T\mu}}{P_i^{M'\mu} P_i^{T'\mu}} = \log \frac{\left(\frac{1}{1-\alpha} + 2\pi'\right) P_i^{M\mu} P_i^{T\mu}}{\left(\frac{1}{1-\alpha} + 2\pi\right) P_i^{M'\mu} P_i^{T'\mu}} = \frac{W_i'}{W_i}$$

as desired. Q.E.D.

Proposition 2. (i) Fix destination country i. Consider firms from all countries j such that $\tau_{ij}^{T'} = \tau_{ij}^{T}$ and $e_{ij}^{T} = 0$, i.e. all country pairs where the firms from source i do not enter destination j pre-reform, and the iceberg costs between them are not affected by the policy change. Then there exists a cutoff x_{i1} for each i such that for firms from countries with cost $c_{ij}^{T} \geq x_{i1}$, the optimal choice is not to enter after the reform ($e_{ij}^{T'} = 0$) while firms from countries with cost $c_{ik}^{T} < x_{i1}$ choose to enter after the reform ($e_{ik}^{T'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$ but $c_{ij}^{T} \geq x_{i1}, c_{ik}^{T} < x_{i1}$. This pair of inequalities imply that $\pi_{ik}^{T} > \pi_{ij}^{T}$. We also know from $e_{ij}^{T} = 0, e_{ik}^{T} = 0$ that

$$\pi_{ij}^{T} < f_{ij}^{T} = \pi_{ij}^{T} + \epsilon$$
$$\pi_{ik}^{T} < f_{ik}^{T} = \pi_{ik}^{T} + \epsilon$$

where $\epsilon > 0$ is a small positive number. Note that

$$\frac{\pi_{ik}^{T'}}{\pi_{ik}^{T}} = \left(\frac{P_i^{T'}}{P_i^{T}}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$
$$\frac{\pi_{ij}^{T'}}{\pi_{ij}^{T}} = \left(\frac{P_i^{T'}}{P_i^{T}}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

So that the change in bilateral gross profits (through the price index and income from dividends) due to the reform for both j and k are the same. Denote this change by δ, i.e.

$$\delta = \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$, we also know that $\pi_{ij}^{T'} \geq f_{ij}^T$ and $\pi_{ik}^{T'} < f_{ik}^T$. Take the first of these two inequalities. We have

$$\pi_{ij}^{T'} \geq f_{ij}^T \quad \Rightarrow \pi_{ij}^{T'} \geq \pi_{ij}^T + \epsilon \quad \Rightarrow (\delta - 1)\pi_{ij}^T \geq \epsilon \quad \Rightarrow \delta - 1 \geq \frac{\epsilon}{\pi_{ij}^T}$$

Then the last inequality implies

$$(\delta - 1)\pi_{ik}^T \geq \frac{\epsilon \pi_{ik}^T}{\pi_{ij}^T} \quad \Rightarrow \pi_{ik}^{T'} - \pi_{ik}^T \geq \frac{\epsilon \pi_{ik}^T}{\pi_{ij}^T} \quad \Rightarrow \pi_{ik}^{T'} \geq \pi_{ik}^T + \frac{\epsilon \pi_{ik}^T}{\pi_{ij}^T}$$

However, combining this with $\pi_{ik}^{T'} < f_{ik}^T = \pi_{ik}^T + \epsilon$ yields

$$\pi_{ik}^T + \frac{\epsilon \pi_{ik}^T}{\pi_{ij}^T} \leq \pi_{ik}^{T'} < f_{ik}^T = \pi_{ik}^T + \epsilon$$

which is a contradiction as $\pi_{ik}^T > \pi_{ij}^T$.

(ii) Fix destination country i. Consider firms from all countries j such that $\tau_{ij}^{T'} = \tau_{ij}^T$ and $e_{ij}^T = 1$, i.e. all country pairs where the firms from source i enter destination j pre-reform, and the iceberg costs between them are not affected by the policy change. Then there exists a cutoff x_{i2} for each i such that for firms from countries with cost $c_{ij}^T \geq x_{i2}$, the optimal choice is not to enter after the reform ($e_{ij}^{T'} = 0$) while firms from countries with cost $c_{ik}^T < x_{i2}$ choose to enter after the reform ($e_{ik}^{T'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$ but $c_{ij}^T \geq x_{i2}, c_{ik}^T < x_{i2}$. This pair of inequalities imply that $\pi_{ik}^T > \pi_{ij}^T$. We also know from $e_{ij}^T = 1, e_{ik}^T = 1$ that

$$\pi_{ij}^T > f_{ij}^T = \epsilon$$
$$\pi_{ik}^T > f_{ik}^T = \epsilon$$

where $\epsilon > 0$ is a small positive number. Note that

$$\frac{\pi_{ik}^{T'}}{\pi_{ik}^T} = \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$
$$\frac{\pi_{ij}^{T'}}{\pi_{ij}^T} = \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

So that the change in bilateral gross profits (through the price index and income from dividends) due to the reform for both j and k are the same. Denote this change by δ, i.e.

$$\delta = \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$, we also know that $\pi_{ij}^{T'} \geq f_{ij}^T$ and $\pi_{ik}^{T'} < f_{ik}^T$. Take the second of these two inequalities. We have

$$\pi_{ik}^{T'} < f_{ik}^T \quad \Rightarrow \pi_{ik}^{T'} < \epsilon \quad \Rightarrow \delta\pi_{ik}^T < \epsilon \quad \Rightarrow \delta < \frac{\epsilon}{\pi_{ik}^T}$$

Then the last inequality implies

$$\delta\pi_{ij}^T < \frac{\epsilon\pi_{ij}^T}{\pi_{ik}^T} \quad \Rightarrow \pi_{ij}^{T'} < \frac{\epsilon\pi_{ij}^T}{\pi_{ik}^T} \quad \Rightarrow \pi_{ij}^{T'} < \epsilon$$

where the last implication comes from $\pi_{ik}^T > \pi_{ij}^T$. This is a contradiction as $\pi_{ij}^{T'} \geq f_{ij}^T = \epsilon$.

(iii) Fix destination country i. Consider firms from all countries j such that $\tau_{ij}^{T'} < \tau_{ij}^T$ and $e_{ij}^T = 1$, i.e. all country pairs where the firms from source i enter destination j pre-reform, and the iceberg costs between them fall as a result of the policy change. Then there exists a cutoff x_{i3} for each i such that for firms from countries with cost $c_{ij}^T \geq x_{i3}$, the optimal choice is not to enter after the reform ($e_{ij}^{T'} = 0$) while firms from countries with cost $c_{ik}^T < x_{i3}$ choose to enter after the reform ($e_{ik}^{T'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$ but $c_{ij}^T \geq x_{i3}, c_{ik}^T < x_{i3}$. This pair of inequalities imply that $\pi_{ik}^T > \pi_{ij}^T$. We also know from $e_{ij}^T = 1, e_{ik}^T = 1$ that

$$\pi_{ij}^T > f_{ij}^T = \epsilon$$
$$\pi_{ik}^T > f_{ik}^T = \epsilon$$

where $\epsilon > 0$ is a small positive number. Note that

$$\frac{\pi_{ik}^{T'}}{\pi_{ik}^T} = \left(\frac{\tau_{ik}^{T'}}{\tau_{ik}^T}\right)^{1-\sigma} \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$
$$\frac{\pi_{ij}^{T'}}{\pi_{ij}^T} = \left(\frac{\tau_{ij}^{T'}}{\tau_{ij}^T}\right)^{1-\sigma} \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Because $\tau_{ik}^{T'}/\tau_{ik}^T = \tau_{ij}^{T'}/\tau_{ij}^T$ the change in bilateral gross profits due to the reform for both j and k are the same. Denote this change by δ, i.e.

$$\delta = \left(\frac{\tau_{ij}^{T'}}{\tau_{ij}^T}\right)^{1-\sigma} \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$, we also know that $\pi_{ij}^{T'} \geq f_{ij}^T$ and $\pi_{ik}^{T'} < f_{ik}^T$. Take the first of these two inequalities. We have

$$\pi_{ik}^{T'} < f_{ik}^T \quad \Rightarrow \pi_{ik}^{T'} < \epsilon \quad \Rightarrow \delta\pi_{ik}^T < \epsilon \quad \Rightarrow \delta < \frac{\epsilon}{\pi_{ik}^T}$$

Then the last inequality implies

$$\delta\pi_{ij}^T < \frac{\epsilon\pi_{ij}^T}{\pi_{ik}^T} \quad \Rightarrow \pi_{ij}^{T'} < \frac{\epsilon\pi_{ij}^T}{\pi_{ik}^T} \quad \Rightarrow \pi_{ij}^{T'} < \epsilon$$

where the last implication comes from $\pi_{ik}^T > \pi_{ij}^T$. This is a contradiction as $\pi_{ij}^{T'} \geq f_{ij}^T = \epsilon$.

(iv) Fix destination country i. Consider firms from all countries j such that $\tau_{ij}^{T'} < \tau_{ij}^T$ and $e_{ij}^T = 0$, i.e. all country pairs where the firms from source i do not enter destination j pre-reform, and the iceberg costs between them fall as a result of the policy change. Then there exists a cutoff x_{i4} for each i such that for firms from countries with cost $c_{ij}^T \geq x_{i4}$, the optimal choice is not to enter after the reform ($e_{ij}^{T'} = 0$) while firms from countries with cost $c_{ik}^T < x_{i4}$ choose to enter after the reform ($e_{ik}^{T'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$ but $c_{ij}^T \geq x_{i4}, c_{ik}^T < x_{i4}$. This pair of inequalities imply that $\pi_{ik}^T > \pi_{ij}^T$. We also know from $e_{ij}^T = 0, e_{ik}^T = 0$ that

$$\pi_{ij}^T < f_{ij}^T = \pi_{ij}^T + \epsilon$$
$$\pi_{ik}^T < f_{ik}^T = \pi_{ik}^T + \epsilon$$

where $\epsilon > 0$ is a small positive number. Note that

$$\frac{\pi_{ik}^{T'}}{\pi_{ik}^T} = \left(\frac{\tau_{ik}^{T'}}{\tau_{ik}^T}\right)^{1-\sigma}\left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1}\frac{Y_i'}{Y_i}$$
$$\frac{\pi_{ij}^{T'}}{\pi_{ij}^T} = \left(\frac{\tau_{ij}^{T'}}{\tau_{ij}^T}\right)^{1-\sigma}\left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1}\frac{Y_i'}{Y_i}$$

Because $\tau_{ik}^{T'}/\tau_{ik}^T = \tau_{ij}^{T'}/\tau_{ij}^T$ the change in bilateral gross profits due to the reform for both j and k are the same. Denote this change by δ, i.e.

$$\delta = \left(\frac{\tau_{ij}^{T'}}{\tau_{ij}^T}\right)^{1-\sigma}\left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1}\frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$, we also know that $\pi_{ij}^{T'} \geq f_{ij}^T$ and $\pi_{ik}^{T'} < f_{ik}^T$. Take the first of these two inequalities. We have

$$\pi_{ij}^{T'} \geq f_{ij}^T \quad \Rightarrow \pi_{ij}^{T'} \geq \pi_{ij}^T + \epsilon \quad \Rightarrow (\delta - 1)\pi_{ij}^T \geq \epsilon \quad \Rightarrow \delta - 1 \geq \frac{\epsilon}{\pi_{ij}^T}$$

Then the last inequality implies

$$(\delta-1)\pi_{ik}^T \geq \frac{\epsilon\pi_{ik}^T}{\pi_{ij}^T} \quad \Rightarrow \pi_{ik}^{T'} - \pi_{ik}^T \geq \frac{\epsilon\pi_{ik}^T}{\pi_{ij}^T} \quad \Rightarrow \pi_{ik}^{T'} \geq \pi_{ik}^T + \frac{\epsilon\pi_{ik}^T}{\pi_{ij}^T}$$

However, combining this with $\pi_{ik}^{T'} < f_{ik}^T = \pi_{ik}^T + \epsilon$ yields

$$\pi_{ik}^T + \frac{\epsilon\pi_{ik}^T}{\pi_{ij}^T} \leq \pi_{ik}^{T'} < f_{ik}^T = \pi_{ik}^T + \epsilon$$

which is a contradiction as $\pi_{ik}^T > \pi_{ij}^T$.

(v) Fix destination country i. Consider firms from all countries j such that $\tau_{ij}^{M'} = \tau_{ij}^M$ and $e_{ij}^M = 0$, i.e. all country pairs where the firms from source i do not enter destination j pre-reform, and the iceberg costs between them are not affected by the policy change. Then there exists a cutoff x_{i5} for each i such that for firms from countries with cost $c_{ij}^M \geq x_{i5}$, the optimal choice is not to enter after the reform ($e_{ij}^{M'} = 0$) while firms from countries with cost $c_{ik}^M < x_{i5}$ choose to enter after the reform ($e_{ik}^{M'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$ but $c_{ij}^M \geq x_{i5}, c_{ik}^M < x_{i5}$. This pair of inequalities imply that $\pi_{ik}^M > \pi_{ij}^M$. We also know from $e_{ij}^M = 0, e_{ik}^M = 0$ that

$$\pi_{ij}^M < f_{ij}^M = \pi_{ij}^M + \epsilon$$
$$\pi_{ik}^M < f_{ik}^M = \pi_{ik}^M + \epsilon$$

where $\epsilon > 0$ is a small positive number. Note that

$$\frac{\pi_{ik}^{M'}}{\pi_{ik}^M} = \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1}\frac{Y_i'}{Y_i}$$
$$\frac{\pi_{ij}^{M'}}{\pi_{ij}^M} = \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1}\frac{Y_i'}{Y_i}$$

So that the change in bilateral gross profits (through the price index and income from dividends) due to the reform for both j and k are the same. Denote this change by δ, i.e.

$$\delta = \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1}\frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$, we also know that $\pi_{ij}^{M'} \geq f_{ij}^M$ and $\pi_{ik}^{M'} < f_{ik}^M$. Take the first of these two inequalities. We have

$$\pi_{ij}^{M'} \geq f_{ij}^M \quad \Rightarrow \pi_{ij}^{M'} \geq \pi_{ij}^M + \epsilon \quad \Rightarrow (\delta-1)\pi_{ij}^M \geq \epsilon \quad \Rightarrow \delta - 1 \geq \frac{\epsilon}{\pi_{ij}^M}$$

Then the last inequality implies

$$(\delta-1)\pi_{ik}^M \geq \frac{\epsilon \pi_{ik}^M}{\pi_{ij}^M} \quad \Rightarrow \pi_{ik}^{M'} - \pi_{ik}^M \geq \frac{\epsilon \pi_{ik}^M}{\pi_{ij}^M} \quad \Rightarrow \pi_{ik}^{M'} \geq \pi_{ik}^M + \frac{\epsilon \pi_{ik}^M}{\pi_{ij}^M}$$

However, combining this with $\pi_{ik}^{M'} < f_{ik}^M = \pi_{ik}^M + \epsilon$ yields

$$\pi_{ik}^M + \frac{\epsilon \pi_{ik}^M}{\pi_{ij}^M} \leq \pi_{ik}^{M'} < f_{ik}^M = \pi_{ik}^M + \epsilon$$

which is a contradiction as $\pi_{ik}^M > \pi_{ij}^M$.

(vi) Fix destination country i. Consider firms from all countries j such that $\tau_{ij}^{M'} = \tau_{ij}^M$ and $e_{ij}^M = 1$, i.e. all country pairs where the firms from source i enter destination j pre-reform, and the iceberg costs between them are not affected by the policy change. Then there exists a cutoff x_{i6} for each i such that for firms from countries with cost $c_{ij}^M \geq x_{i6}$, the optimal choice is not to enter after the reform ($e_{ij}^{M'} = 0$) while firms from countries with cost $c_{ik}^M < x_{i6}$ choose to enter after the reform ($e_{ik}^{M'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$ but $c_{ij}^M \geq x_{i6}, c_{ik}^M < x_{i6}$. This pair of inequalities imply that $\pi_{ik}^M > \pi_{ij}^M$. We also know from $e_{ij}^M = 1, e_{ik}^M = 1$ that

$$\pi_{ij}^M > f_{ij}^M = \epsilon$$
$$\pi_{ik}^M > f_{ik}^M = \epsilon$$

where $\epsilon > 0$ is a small positive number. Note that

$$\frac{\pi_{ik}^{M'}}{\pi_{ik}^M} = \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$
$$\frac{\pi_{ij}^{M'}}{\pi_{ij}^M} = \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

So that the change in bilateral gross profits (through the price index and income from dividends) due to the reform for both j and k are the same. Denote this change by δ, i.e.

$$\delta = \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$, we also know that $\pi_{ij}^{M'} \geq f_{ij}^M$ and $\pi_{ik}^{M'} < f_{ik}^M$. Take the second of these two inequalities. We have

$$\pi_{ik}^{M'} < f_{ik}^M \quad \Rightarrow \pi_{ik}^{M'} < \epsilon \quad \Rightarrow \delta \pi_{ik}^M < \epsilon \quad \Rightarrow \delta < \frac{\epsilon}{\pi_{ik}^M}$$

Then the last inequality implies

$$\delta \pi_{ij}^{M} < \frac{\epsilon \pi_{ij}^{M}}{\pi_{ik}^{M}} \quad \Rightarrow \pi_{ij}^{M'} < \frac{\epsilon \pi_{ij}^{M}}{\pi_{ik}^{M}} \quad \Rightarrow \pi_{ij}^{M'} < \epsilon$$

where the last implication comes from $\pi_{ik}^{M} > \pi_{ij}^{M}$. This is a contradiction as $\pi_{ij}^{M'} \geq f_{ij}^{M} = \epsilon$.

(vii) Fix destination country i. Consider firms from all countries j such that $\tau_{ij}^{M'} < \tau_{ij}^{M}$ and $e_{ij}^{M} = 1$, i.e. all country pairs where the firms from source i enter destination j pre-reform, and the iceberg costs between them fall as a result of the policy change. Then there exists a cutoff x_{i7} for each i such that for firms from countries with cost $c_{ij}^{M} \geq x_{i7}$, the optimal choice is not to enter after the reform ($e_{ij}^{M'} = 0$) while firms from countries with cost $c_{ik}^{M} < x_{i7}$ choose to enter after the reform ($e_{ik}^{M'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$ but $c_{ij}^{M} \geq x_{i7}, c_{ik}^{M} < x_{i7}$. This pair of inequalities imply that $\pi_{ik}^{M} > \pi_{ij}^{M}$. We also know from $e_{ij}^{M} = 1, e_{ik}^{M} = 1$ that

$$\begin{aligned} \pi_{ij}^{M} &> f_{ij}^{M} = \epsilon \\ \pi_{ik}^{M} &> f_{ik}^{M} = \epsilon \end{aligned}$$

where $\epsilon > 0$ is a small positive number. Note that

$$\begin{aligned} \frac{\pi_{ik}^{M'}}{\pi_{ik}^{M}} &= \left(\frac{\tau_{ik}^{M'}}{\tau_{ik}^{M}}\right)^{1-\sigma} \left(\frac{P_i^{M'}}{P_i^{M}}\right)^{\sigma-1} \frac{Y_i'}{Y_i} \\ \frac{\pi_{ij}^{M'}}{\pi_{ij}^{M}} &= \left(\frac{\tau_{ij}^{M'}}{\tau_{ij}^{M}}\right)^{1-\sigma} \left(\frac{P_i^{M'}}{P_i^{M}}\right)^{\sigma-1} \frac{Y_i'}{Y_i} \end{aligned}$$

Because $\tau_{ik}^{M'}/\tau_{ik}^{M} = \tau_{ij}^{M'}/\tau_{ij}^{M}$ the change in bilateral gross profits due to the reform for both j and k are the same. Denote this change by δ, i.e.

$$\delta = \left(\frac{\tau_{ij}^{M'}}{\tau_{ij}^{M}}\right)^{1-\sigma} \left(\frac{P_i^{M'}}{P_i^{M}}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$, we also know that $\pi_{ij}^{M'} \geq f_{ij}^{M}$ and $\pi_{ik}^{M'} < f_{ik}^{M}$. Take the first of these two inequalities. We have

$$\pi_{ik}^{M'} < f_{ik}^{M} \quad \Rightarrow \pi_{ik}^{M'} < \epsilon \quad \Rightarrow \delta \pi_{ik}^{M} < \epsilon \quad \Rightarrow \delta < \frac{\epsilon}{\pi_{ik}^{M}}$$

Then the last inequality implies

$$\delta \pi_{ij}^{M} < \frac{\epsilon \pi_{ij}^{M}}{\pi_{ik}^{M}} \quad \Rightarrow \pi_{ij}^{M'} < \frac{\epsilon \pi_{ij}^{M}}{\pi_{ik}^{M}} \quad \Rightarrow \pi_{ij}^{M'} < \epsilon$$

where the last implication comes from $\pi_{ik}^M > \pi_{ij}^M$. This is a contradiction as $\pi_{ij}^{M'} \geq f_{ij}^M = \epsilon$.

(viii) Fix destination country i. Consider firms from all countries j such that $\tau_{ij}^{M'} < \tau_{ij}^M$ and $e_{ij}^M = 0$, i.e. all country pairs where the firms from source i do not enter destination j pre-reform, and the iceberg costs between them fall as a result of the policy change. Then there exists a cutoff x_{i8} for each i such that for firms from countries with cost $c_{ij}^M \geq x_{i8}$, the optimal choice is not to enter after the reform ($e_{ij}^{M'} = 0$) while firms from countries with cost $c_{ik}^M < x_{i8}$ choose to enter after the reform ($e_{ik}^{M'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$ but $c_{ij}^M \geq x_{i8}, c_{ik}^M < x_{i8}$. This pair of inequalities imply that $\pi_{ik}^M > \pi_{ij}^M$. We also know from $e_{ij}^M = 0, e_{ik}^M = 0$ that

$$\pi_{ij}^M < f_{ij}^M = \pi_{ij}^M + \epsilon$$
$$\pi_{ik}^M < f_{ik}^M = \pi_{ik}^M + \epsilon$$

where $\epsilon > 0$ is a small positive number. Note that

$$\frac{\pi_{ik}^{M'}}{\pi_{ik}^M} = \left(\frac{\tau_{ik}^{M'}}{\tau_{ik}^M}\right)^{1-\sigma} \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$
$$\frac{\pi_{ij}^{M'}}{\pi_{ij}^M} = \left(\frac{\tau_{ij}^{M'}}{\tau_{ij}^M}\right)^{1-\sigma} \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Because $\tau_{ik}^{M'}/\tau_{ik}^M = \tau_{ij}^{M'}/\tau_{ij}^M$ the change in bilateral gross profits due to the reform for both j and k are the same. Denote this change by δ, i.e.

$$\delta = \left(\frac{\tau_{ij}^{M'}}{\tau_{ij}^M}\right)^{1-\sigma} \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$, we also know that $\pi_{ij}^{M'} \geq f_{ij}^M$ and $\pi_{ik}^{M'} < f_{ik}^M$. Take the first of these two inequalities. We have

$$\pi_{ij}^{M'} \geq f_{ij}^M \quad \Rightarrow \pi_{ij}^{M'} \geq \pi_{ij}^M + \epsilon \quad \Rightarrow (\delta - 1)\pi_{ij}^M \geq \epsilon \quad \Rightarrow \delta - 1 \geq \frac{\epsilon}{\pi_{ij}^M}$$

Then the last inequality implies

$$(\delta - 1)\pi_{ik}^M \geq \frac{\epsilon \pi_{ik}^M}{\pi_{ij}^M} \quad \Rightarrow \pi_{ik}^{M'} - \pi_{ik}^M \geq \frac{\epsilon \pi_{ik}^M}{\pi_{ij}^M} \quad \Rightarrow \pi_{ik}^{M'} \geq \pi_{ik}^M + \frac{\epsilon \pi_{ik}^M}{\pi_{ij}^M}$$

However, combining this with $\pi_{ik}^{M'} < f_{ik}^M = \pi_{ik}^M + \epsilon$ yields

$$\pi_{ik}^M + \frac{\epsilon \pi_{ik}^M}{\pi_{ij}^M} \leq \pi_{ik}^{M'} < f_{ik}^M = \pi_{ik}^M + \epsilon$$

which is a contradiction as $\pi_{ik}^M > \pi_{ij}^M$.

3 Appendix B: Computational Algorithm

Step 1: Parameterize the distribution of decision rules e'_{ij} for all i, j as follows:

$$e_{ij}^{T'} = \begin{cases} 0 & \text{if } \tau_{ij}^{T'} = \tau_{ij}^{T},\ e_{ij}^{T} = 0,\ c_{ij}^{T} \geq x'_{i1} \\ 1 & \text{if } \tau_{ij}^{T'} = \tau_{ij}^{T},\ e_{ij}^{T} = 0,\ c_{ij}^{T} < x'_{i1} \\ 0 & \text{if } \tau_{ij}^{T'} = \tau_{ij}^{T},\ e_{ij}^{T} = 1,\ c_{ij}^{T} \geq x'_{i2} \\ 1 & \text{if } \tau_{ij}^{T'} = \tau_{ij}^{T},\ e_{ij}^{T} = 1,\ c_{ij}^{T} < x'_{i2} \\ 0 & \text{if } \tau_{ij}^{T'} < \tau_{ij}^{T},\ e_{ij}^{T} = 1,\ c_{ij}^{T} \geq x'_{i3} \\ 1 & \text{if } \tau_{ij}^{T'} < \tau_{ij}^{T},\ e_{ij}^{T} = 1,\ c_{ij}^{T} < x'_{i3} \\ 0 & \text{if } \tau_{ij}^{T'} < \tau_{ij}^{T},\ e_{ij}^{T} = 0,\ c_{ij}^{T} \geq x'_{i4} \\ 1 & \text{if } \tau_{ij}^{T'} < \tau_{ij}^{T},\ e_{ij}^{T} = 0,\ c_{ij}^{T} < x'_{i4} \end{cases}$$

$$e_{ij}^{M'} = \begin{cases} 0 & \text{if } \tau_{ij}^{M'} = \tau_{ij}^{M},\ e_{ij}^{M} = 0,\ c_{ij}^{M} \geq x'_{i5} \\ 1 & \text{if } \tau_{ij}^{M'} = \tau_{ij}^{M},\ e_{ij}^{M} = 0,\ c_{ij}^{M} < x'_{i5} \\ 0 & \text{if } \tau_{ij}^{M'} = \tau_{ij}^{M},\ e_{ij}^{M} = 1,\ c_{ij}^{M} \geq x'_{i6} \\ 1 & \text{if } \tau_{ij}^{M'} = \tau_{ij}^{M},\ e_{ij}^{M} = 1,\ c_{ij}^{M} < x'_{i6} \\ 0 & \text{if } \tau_{ij}^{M'} < \tau_{ij}^{M},\ e_{ij}^{M} = 1,\ c_{ij}^{M} \geq x'_{i7} \\ 1 & \text{if } \tau_{ij}^{M'} < \tau_{ij}^{M},\ e_{ij}^{M} = 1,\ c_{ij}^{M} < x'_{i7} \\ 0 & \text{if } \tau_{ij}^{M'} < \tau_{ij}^{M},\ e_{ij}^{M} = 0,\ c_{ij}^{M} \geq x'_{i8} \\ 1 & \text{if } \tau_{ij}^{M'} < \tau_{ij}^{M},\ e_{ij}^{M} = 0,\ c_{ij}^{M} < x'_{i8} \end{cases}$$

Note the similarity between these decision rules and the lemmata proved earlier.

Step 2: Guess an initial vector of cutoffs $\{(x_{i1}, x_{i2}, x_{i3}, x_{i4}, x_{i5}, x_{i6}, x_{i7}, x_{i8})\}_{i=1,2,\ldots,N}$. Do this with percentiles of the elements in the $8N$ sets $\{\Omega_{i1}, \Omega_{i2}, \Omega_{i3}, \Omega_{i4}, \Omega_{i5}, \Omega_{i6}, \Omega_{i7}, \Omega_{i8}\}_{i=1,2,\ldots,N}$, defined by

$$\begin{aligned} \Omega_{i1} &= \{j : \tau_{ij}^{T'} = \tau_{ij}^{T},\ e_{ij}^{T} = 0\} \\ \Omega_{i2} &= \{j : \tau_{ij}^{T'} = \tau_{ij}^{T},\ e_{ij}^{T} = 1\} \\ \Omega_{i3} &= \{j : \tau_{ij}^{T'} < \tau_{ij}^{T},\ e_{ij}^{T} = 1\} \\ \Omega_{i4} &= \{j : \tau_{ij}^{T'} < \tau_{ij}^{T},\ e_{ij}^{T} = 0\} \\ \Omega_{i5} &= \{j : \tau_{ij}^{M'} = \tau_{ij}^{M},\ e_{ij}^{M} = 0\} \\ \Omega_{i6} &= \{j : \tau_{ij}^{M'} = \tau_{ij}^{M},\ e_{ij}^{M} = 1\} \\ \Omega_{i7} &= \{j : \tau_{ij}^{M'} < \tau_{ij}^{M},\ e_{ij}^{M} = 1\} \\ \Omega_{i8} &= \{j : \tau_{ij}^{M'} < \tau_{ij}^{M},\ e_{ij}^{M} = 0\} \end{aligned}$$

A cutoff x_{ih} is then associated with the percentile $k_{ih} = 100|\hat{\Omega}_{ih}|/|\Omega_{ih}|$ for $h = 1, 2, 3, \ldots, 8$, where $\hat{\Omega}_{ih} = \{j : j \in \Omega_{ih},\ c_{ij} < x_{ih}\}$.

Step 3: Given the parameterization of the decision rules and guess for the cutoffs, we can then compute what the prices indices P_i^t are for all i as well as the dividend per share π^t. Here the superscript t denotes the t-th iteration.

Step 4: Given the price indices P_i^t and dividend per share π^t, we can compute the decision rule e'_{ij} for firms from each source country j and destination country i. Denote this matrix (i.e. distribution) of decision rules by E. These decision rules then yield price indices P_i^n and dividend per share π^n.

Step 5: Given the price indices P_i^n and dividend per share π^n, we can construct the profits π_{ij}^n that firms from j can make by exporting to i and compare this against the corresponding fixed cost f_{ij} to determine the entry pattern $e_{ij}^{'n}$. Denote the matrix of such entry patterns by E^n. If $E = E^n$ then we are done.

Step 6: If $E \neq E^n$, check if the cutoff rules implied by the decision rules just obtained k_{ih}^n coincide with the guess for the cutoff rules k_{ih}^t. If yes, then terminate; in this case we obtain an approximate equilibrium as no further updating can be done and $E \neq E^n$. If no, then proceed to update the cutoffs according to $k_{ih}^{t+1} = \lambda k_{ih}^n + (1-\lambda)k_{ih}^t$ where the step size is $\lambda \in (0,1)$. The cutoff rules implied by the decision rules e'_{ij} are constructed as follows: $k_{ih}^n = 100|\hat{\Omega}_{ih}^n|/|\Omega_{ih}|$ where

$$
\begin{aligned}
\hat{\Omega}_{i1}^n &= \{j : j \in \Omega_{i1},\ e_{ij}^{T'} = 1\} \\
\hat{\Omega}_{i2}^n &= \{j : j \in \Omega_{i2},\ e_{ij}^{T'} = 1\} \\
\hat{\Omega}_{i3}^n &= \{j : j \in \Omega_{i3},\ e_{ij}^{T'} = 1\} \\
\hat{\Omega}_{i4}^n &= \{j : j \in \Omega_{i4},\ e_{ij}^{T'} = 1\} \\
\hat{\Omega}_{i5}^n &= \{j : j \in \Omega_{i5},\ e_{ij}^{M'} = 1\} \\
\hat{\Omega}_{i6}^n &= \{j : j \in \Omega_{i6},\ e_{ij}^{M'} = 1\} \\
\hat{\Omega}_{i7}^n &= \{j : j \in \Omega_{i7},\ e_{ij}^{M'} = 1\} \\
\hat{\Omega}_{i8}^n &= \{j : j \in \Omega_{i8},\ e_{ij}^{M'} = 1\}
\end{aligned}
$$

With the updated cutoff rules, return to Step 3 and iterate until convergence.

4 Appendix C: Two-Country Example

4.1 Introduction

There are two countries $i = 1, 2$. Each has a measure $\theta_i = w_i L_i$ of firms that are considering which markets to enter. For simplicity consider only an environment with trade (i.e. no multinational production). International trade is subject to both iceberg and fixed costs, which can be asymmetric across country pairs. Consider a policy reform that lowers the iceberg costs for firms from 2 to export to 1 from $\tau_{12} = 2$ to $\tau'_{12} = 1$. Let $\tau_{21} = \tau'_{21} = 1$. The trade elasticity is $\sigma = 2$ and the expenditure share for the differentiated goods sector is

$\mu = 1/2$. The other exogenous parameters are given below

$$w_1 = 1, \quad w_2 = 1, \quad L_1 = 1, \quad L_2 = 1, \quad \phi_1 = 2, \quad \phi_2 = 1$$

This implies

$$p_{11} = p'_{11} = 1, \quad p_{22} = p'_{22} = 2, \quad p_{12} = \frac{\sigma}{\sigma - 1}\frac{w_2\tau_{12}}{\phi_2} = 4, \quad p_{21} = \frac{\sigma}{\sigma - 1}\frac{w_1\tau_{21}}{\phi_1} = 1, \quad p'_{12} = \frac{\sigma}{\sigma - 1}\frac{w_2\tau'_{12}}{\phi_2} = 2$$

Now we will look at the equilibria that can arise given different values for (f_{12}, f_{21}), i.e. we will partition the $f_{12} - f_{21}$ space into the regions where the four cases $(e_{12}, e_{21}) \in \{(0,0), (0,1), (1,0), (1,1)\}$ apply. We will do this both pre-reform and post-reform. Note: In general, one should consider not only four cases, but sixteen cases, as entry decisions e_{11}, e_{22} are also binary choice variables that can be 0 or 1. To ease exposition, I assume throughout this illustration that these decisions are $e_{11} = 1$ and $e_{22} = 1$.

Note that given our assumptions on the values of the parameters, the profit for a firm from j exporting to i denoted by π_{ij} is simply given by

$$\pi_{ij} = \mu\frac{1}{\sigma}\left(\frac{p_{ij}}{P_i}\right)^{1-\sigma} w_i L_i (1+\pi) = \frac{1}{2}\frac{1}{2}\left(\frac{p_{ij}}{P_i}\right)^{-1}(1+\pi) = \frac{1}{4}\frac{P_i}{p_{ij}}(1+\pi)$$

This will be useful for the characterization that follows.

4.2 Pre-Reform

4.2.1 Case 1: $(e_{12}, e_{21}) = (0, 0)$

In this equilibrium, firms only operate domestically and there is no entry into international markets either way. Hence we have

$$P_1 = p_{11} = 1, \quad P_2 = p_{22} = 2$$

In this case we also have $\pi = \frac{1}{3}$, and

$$\begin{aligned}
\pi_{11} &= \frac{1}{4}(1+\pi) \\
\pi_{12} &= \frac{1}{4}\frac{P_1}{p_{12}}(1+\pi) = \frac{1}{16}(1+\pi) = \frac{1}{16}\frac{4}{3} = \frac{1}{12} \leq f_{12} \\
\pi_{21} &= \frac{1}{4}\frac{P_2}{p_{21}}(1+\pi) = \frac{1}{2}(1+\pi) = \frac{2}{3} \leq f_{21} \\
\pi_{22} &= \frac{1}{4}\frac{P_2}{p_{22}} = \frac{1}{4}(1+\pi) = \frac{1}{3}
\end{aligned}$$

4.2.2 Case 2: $(e_{12}, e_{21}) = (0, 1)$

In this equilibrium, country 2 firms enter (i.e. exports to) country 1 but not the other way around. Hence we have

$$P_1 = p_{11} = 1, \quad P_2 = [p_{21}^{-1} + p_{22}^{-1}]^{-1} = \frac{2}{3}$$

In this case we have

$$\begin{aligned}
\pi_{11} &= \frac{1}{4}(1+\pi) \\
\pi_{12} &= \frac{1}{16}(1+\pi) \\
\pi_{21} &= \frac{1}{4}\frac{2/3}{1}(1+\pi) = \frac{1}{6}(1+\pi) \\
\pi_{22} &= \frac{1}{4}\frac{2/3}{2} = \frac{1}{12}(1+\pi)
\end{aligned}$$

Hence

$$\pi = \frac{\frac{3+2+1}{12}(1+\pi) - f_{21}}{2} = \frac{1}{3} - \frac{2}{3}f_{21}$$

It is instructive to note that

$$\begin{aligned}
\pi_{\max} &= \frac{1}{3}, \quad f_{21} = 0 \\
\pi_{\min} &= \frac{1}{6}(1+\pi_{\min}) \Rightarrow \pi_{\min} = \frac{1}{5}, \quad f_{21} = \frac{1}{5}
\end{aligned}$$

In general

$$\pi_{21} = \frac{1}{16}\Big(\frac{4}{3} - \frac{2}{3}f_{21}\Big) = \frac{1}{12} - \frac{1}{24}f_{21} \leq f_{12}, \text{for} f_{21} \in (0, \frac{1}{5})$$

4.2.3 Case 3: $(e_{12}, e_{21}) = (1, 0)$

In this equilibrium, country 1 firms enter (i.e. exports to) country 2 but not the other way around. Hence we have

$$P_1 = [p_{11}^{-1} + p_{12}^{-1}]^{-1} = \frac{4}{5}, \quad P_2 = p_{22} = 2$$

In this case we have

$$\pi_{11} = \frac{1}{4}\frac{P_1}{p_{11}}(1+\pi) = \frac{1}{4}\frac{4}{5}(1+\pi) = \frac{1}{5}(1+\pi)$$
$$\pi_{12} = \frac{1}{4}\frac{P_1}{p_{12}}(1+\pi) = \frac{1}{4}\frac{4/5}{4}(1+\pi) = \frac{1}{20}(1+\pi)$$
$$\pi_{21} = \frac{1}{4}\frac{P_2}{p_{21}}(1+\pi) = \frac{1}{4}\frac{2}{1}(1+\pi) = \frac{1}{2}(1+\pi)$$
$$\pi_{22} = \frac{1}{4}\frac{P_2}{p_{22}}(1+\pi) = \frac{1}{4}(1+\pi)$$

Hence

$$\pi = \frac{\frac{5+4+1}{20}(1+\pi) - f_{12}}{2} = \frac{1}{3} - \frac{2}{3}f_{12}$$

It is instructive to note that

$$\pi_{\max} = \frac{1}{3}, \quad f_{12} = 0$$
$$\pi_{\min} = \frac{\frac{5+4}{20}(1+\pi)}{2} = \frac{9}{40}(1+\pi_{\min}) \Rightarrow \pi_{\min} = \frac{9}{31}, \quad f_{12} = \frac{2}{31}$$

In general

$$\pi_{21} = \frac{1}{2}\Big(\frac{4}{3} - \frac{2}{3}f_{12}\Big) = \frac{2}{3} - \frac{1}{3}f_{12} \le f_{21}, \mathrm{for} f_{12} \in (0, \frac{2}{13})$$

4.2.4 Case 4: $(e_{12}, e_{21}) = (1, 1)$

In this equilibrium, country 1 firms enter (i.e. exports to) country 2 and vice versa. Hence we have

$$P_1 = [p_{11}^{-1} + p_{12}^{-1}]^{-1} = \frac{4}{5}, \quad P_2 = [p_{21}^{-1} + p_{22}^{-1}]^{-1} = \frac{2}{3}$$

In this case we have

$$\pi_{11} = \frac{1}{5}(1+\pi)$$
$$\pi_{12} = \frac{1}{20}(1+\pi)$$
$$\pi_{21} = \frac{1}{6}(1+\pi)$$
$$\pi_{22} = \frac{1}{12}(1+\pi)$$

Hence

$$\pi = \frac{1}{3} - \frac{2}{3}(f_{12} + f_{21})$$

$$\pi_{\max} = \frac{1}{3}, \quad f_{12} = 0, \quad f_{21} = 0$$

$$\pi_{\min} = \frac{\frac{5+12}{60}(1+\pi)}{2} = \frac{17}{120}(1+\pi_{\min}) \Rightarrow \pi_{\min} = \frac{17}{103}, \quad f_{21} = \frac{1}{20}\frac{120}{103} = \frac{6}{103}, \quad f_{12} = \frac{1}{6}\frac{120}{103} = \frac{20}{103}$$

In general

$$\pi_{12} = \frac{1}{20}\Big(\frac{4}{3} - \frac{2}{3}f_{12} - \frac{2}{3}f_{21}\Big) = \frac{2}{30} - \frac{1}{30}f_{12} - \frac{1}{30}f_{21} \leq f_{12}$$
$$\pi_{21} = \frac{1}{6}\Big(\frac{4}{3} - \frac{2}{3}f_{12} - \frac{2}{3}f_{21}\Big) = \frac{2}{9} - \frac{1}{9}f_{12} - \frac{1}{9}f_{21} \leq f_{21}$$

This implies

$$f_{12} \leq \frac{30}{31}\Big(\frac{2}{30} - \frac{1}{30}f_{21}\Big)$$
$$f_{21} \leq \frac{9}{10}\Big(\frac{2}{9} - \frac{1}{9}f_{12}\Big)$$

Hence the endpoints are $\{(\frac{2}{31}, 0), (0, \frac{2}{10})\}$.

4.3 Post-Reform

4.3.1 Case 1: $(e'_{12}, e'_{21}) = (0, 0)$

In this equilibrium, firms only operate domestically and there is no entry into international markets either way. Hence we have

$$P'_1 = p'_{11} = 1, \quad P'_2 = p'_{22} = 2$$

In this case we also have $\pi' = \frac{1}{3}$, and

$$\pi'_{11} = \frac{1}{4}(1+\pi')$$
$$\pi'_{12} = \frac{1}{4}\frac{P'_1}{p'_{12}}(1+\pi') = \frac{1}{8}(1+\pi') = \frac{1}{8}\frac{4}{3} = \frac{1}{6} \leq f_{12}$$
$$\pi'_{21} = \frac{1}{4}\frac{P'_2}{p'_{21}}(1+\pi') = \frac{1}{2}(1+\pi') = \frac{2}{3} \leq f_{21}$$
$$\pi'_{22} = \frac{1}{4}\frac{P'_2}{p'_{22}} = \frac{1}{4}(1+\pi') = \frac{1}{3}$$

4.3.2 Case 2: $(e'_{12}, e'_{21}) = (0, 1)$

In this equilibrium, country 2 firms enter (i.e. exports to) country 1 but not the other way around. Hence we have

$$P'_1 = p'_{11} = 1, \quad P'_2 = [p'^{-1}_{21} + p'^{-1}_{22}]^{-1} = \frac{2}{3}$$

In this case we have

$$\pi'_{11} = \frac{1}{4}(1+\pi')$$
$$\pi'_{12} = \frac{1}{8}(1+\pi')$$
$$\pi'_{21} = \frac{1}{6}(1+\pi')$$
$$\pi'_{22} = \frac{1}{12}(1+\pi')$$

Hence

$$\pi' = \frac{1}{3} - \frac{2}{3}f_{21}$$

It is instructive to note that

$$\pi'_{\max} = \frac{1}{3}, \quad f_{21} = 0$$
$$\pi'_{\min} = \frac{1}{5}, \quad f_{21} = \frac{1}{5}$$

In general

$$\pi'_{21} = \frac{1}{8}\Big(\frac{4}{3} - \frac{2}{3}f_{21}\Big) = \frac{1}{6} - \frac{1}{12}f_{21} \leq f_{12}, \text{for} f_{21} \in (0, \frac{1}{5})$$

4.3.3 Case 3: $(e'_{12}, e'_{21}) = (1, 0)$

In this equilibrium, country 1 firms enter (i.e. exports to) country 2 but not the other way around. Hence we have

$$P'_1 = [p'^{-1}_{11} + p'^{-1}_{12}]^{-1} = \frac{2}{3}, \quad P'_2 = p'_{22} = 2$$

In this case we have

$$\pi'_{11} = \frac{1}{6}(1+\pi')$$
$$\pi'_{12} = \frac{1}{12}(1+\pi')$$
$$\pi'_{21} = \frac{1}{2}(1+\pi')$$
$$\pi'_{22} = \frac{1}{4}(1+\pi')$$

Hence

$$\pi' = \frac{\frac{2+1+13}{12}(1+\pi') - f_{12}}{2} = \frac{1}{3} - \frac{2}{3}f_{12}$$

It is instructive to note that

$$\pi'_{\max} = \frac{1}{3}, \quad f_{12} = 0$$
$$\pi'_{\min} = \frac{5}{24}(1+\pi'_{\min}) \Rightarrow \pi'_{\min} = \frac{5}{19}, \quad f_{12} = \frac{2}{19}$$

In general

$$\pi'_{21} = \frac{1}{2}\Big(\frac{4}{3} - \frac{2}{3}f_{12}\Big) = \frac{2}{3} - \frac{1}{3}f_{12} \leq f_{21}, \mathrm{for} f_{12} \in (0, \frac{2}{19})$$

4.3.4 Case 4: $(e_{12}, e_{21}) = (1, 1)$

In this equilibrium, country 1 firms enter (i.e. exports to) country 2 and vice versa. Hence we have

$$P'_1 = [p'^{-1}_{11} + p'^{-1}_{12}]^{-1} = \frac{2}{3}, \quad P'_2 = [p'^{-1}_{21} + p'^{-1}_{22}]^{-1} = \frac{2}{3}$$

In this case we have

$$\pi'_{11} = \frac{1}{6}(1+\pi')$$
$$\pi'_{12} = \frac{1}{12}(1+\pi')$$
$$\pi'_{21} = \frac{1}{6}(1+\pi')$$
$$\pi'_{22} = \frac{1}{12}(1+\pi')$$

Hence

$$\pi' = \frac{1}{3} - \frac{2}{3}(f_{12} + f_{21})$$

$$\pi'_{\max} = \frac{1}{3}, \quad f_{12} = 0, \quad f_{21} = 0$$
$$\pi'_{\min} = \frac{1}{8}(1+\pi'_{\min}) \Rightarrow \pi'_{\min} = \frac{1}{7}, \quad f_{21} = \frac{1}{6}\frac{8}{7} = \frac{4}{21}, \quad f_{12} = \frac{1}{12}\frac{8}{7} = \frac{2}{21}$$

In general

$$\pi'_{12} = \frac{1}{12}\Big(\frac{4}{3} - \frac{2}{3}f_{12} - \frac{2}{3}f_{21}\Big) = \frac{1}{9} - \frac{1}{18}f_{12} - \frac{1}{18}f_{21} \leq f_{12}$$
$$\pi'_{21} = \frac{1}{6}\Big(\frac{4}{3} - \frac{2}{3}f_{12} - \frac{2}{3}f_{21}\Big) = \frac{2}{9} - \frac{1}{9}f_{12} - \frac{1}{9}f_{21} \leq f_{21}$$

This implies

$$f_{12} \leq \frac{18}{19}\Big(\frac{1}{9} - \frac{1}{18}f_{21}\Big)$$
$$f_{21} \leq \frac{9}{10}\Big(\frac{2}{9} - \frac{1}{9}f_{12}\Big)$$

Hence the endpoints are $\{(\frac{2}{19}, 0), (0, \frac{2}{10})\}$.

4.4 Equilibria Pre- and Post-Reform

Given the characterization of the two previous sections, we can partition the $f_{12} - f_{21}$ space into the following regions.

Pre-reform, we have the five regions $P_i, \quad i = 0, 1, \ldots, 4$ where P_i corresponds to the region that applies to the i-th case described earlier (e.g. P_2 corresponds to $(e_{12}, e_{21}) = (0, 1)$), and the 0-th case is the region in the parameter space where no equilibria exist.

$$
\begin{aligned}
&P_1 = \{(f_{12}, f_{21}) \in R_+^2 : \quad f_{12} \geq 1/12, \quad f_{21} \geq 2/3\} \\
&P_2 = \{(f_{12}, f_{21}) \in R_+^2 : \quad f_{12} \geq 1/12 - 1/24 f_{12}, \quad f_{21} \leq 1/5\} \\
&P_3 = \{(f_{12}, f_{21}) \in R_+^2 : \quad f_{21} \geq 2/3 - 1/3 f_{12}, \quad f_{12} \leq 2/31\} \\
&P_4 = \{(f_{12}, f_{21}) \in R_+^2 : \quad f_{12} \geq 30/31(2/30 - 1/30 f_{12}), \quad f_{21} \geq 9/10(2/9 - 1/9 f_{12})\} \\
&P_0 = R_+^2 \setminus \{P_1 \cup P_2 \cup P_3 \cup P_4\}
\end{aligned}
$$

Post-reform, the fixed cost space is partitioned into the following regions

$$
\begin{aligned}
&Q_1 = \{(f_{12}, f_{21}) \in R_+^2 : \quad f_{12} \geq 1/6, \quad f_{21} \geq 2/3\} \\
&Q_2 = \{(f_{12}, f_{21}) \in R_+^2 : \quad f_{12} \geq 1/6 - 1/12 f_{12}, \quad f_{21} \leq 1/5\} \\
&Q_3 = \{(f_{12}, f_{21}) \in R_+^2 : \quad f_{21} \geq 2/3 - 1/3 f_{12}, \quad f_{12} \leq 2/19\} \\
&Q_4 = \{(f_{12}, f_{21}) \in R_+^2 : \quad f_{12} \geq 18/19(1/9 - 1/18 f_{12}), \quad f_{21} \geq 9/10(2/9 - 1/9 f_{12})\} \\
&Q_0 = R_+^2 \setminus \{Q_1 \cup Q_2 \cup Q_3 \cup Q_4\}
\end{aligned}
$$

Then combining these two partitions of the fixed-cost space, we get the following regions:

$$
\begin{aligned}
&A_{11} = P_1 \cap Q_1 = \{(f_{12}, f_{21}) \in R_+^2 : \quad f_{12} \geq 1/6, \quad f_{21} \geq 2/3\} \\
&A_{13} = P_1 \cap Q_3 = \{(f_{12}, f_{21}) \in R_+^2 : \quad 1/12 \leq f_{12} \leq 2/19, \quad f_{21} \geq 2/3\} \\
&A_{33} = P_3 \cap Q_3 = \{(f_{12}, f_{21}) \in R_+^2 : \quad f_{21} \geq 2/3 - 1/3 f_{12}, \quad f_{12} \leq 2/31\} \\
&A_{22} = P_2 \cap Q_2 = \{(f_{12}, f_{21}) \in R_+^2 : \quad f_{12} \geq 1/6 - 1/12 f_{12}, \quad f_{21} \leq 1/5\} \\
&A_{24} = P_2 \cap Q_4 = \{(f_{12}, f_{21}) \in R_+^2 : \quad f_{12} \geq 1/12 - 1/24 f_{21}, \quad f_{21} \leq 1/5\} \\
&\qquad \cap \{(f_{12}, f_{21}) : f_{12} \leq 18/19(1/9 - 1/18 f_{21}), \quad f_{21} \leq 9/10(2/9 - 1/9 f_{12})\} \\
&A_{44} = P_4 \cap Q_4 = \{(f_{12}, f_{21}) \in R_+^2 : \quad f_{21} \leq 9/10(2/9 - 1/9 f_{12}), \quad f_{12} \geq 30/31(2/30 - 1/30 f_{12})\} \\
&A_{10} = P_1 \cap Q_0 = \{(f_{12}, f_{21}) \in R_+^2 : \quad 2/19 \leq f_{12} \leq 1/6, \quad f_{21} \geq 2/3\} \\
&A_{20} = P_2 \cap Q_0 = P_2 \setminus \{A_{24} \cup A_{22}\} \\
&A_{03} = P_0 \cap Q_3 = Q_3 \setminus \{A_{33} \cup A_{13}\} \\
&A_{04} = P_0 \cap Q_4 = Q_4 \setminus \{A_{44} \cup A_{24}\} \\
&A_{12} = A_{14} = A_{21} = A_{23} = A_{31} = A_{32} = A_{34} = A_{41} = A_{42} = A_{43} = \{\} \\
&A_{01} = A_{02} = A_{30} = A_{40} = \{\} \\
&A_{00} = R_+^2 \setminus \{A_{11} \cup A_{13} \cup A_{33} \cup A_{22} \cup A_{24} \cup A_{44} \cup A_{10} \cup A_{20} \cup A_{03} \cup A_{04}\}
\end{aligned}
$$

Here A_{ij} means that pre-reform, the equilibrium is Case i and post-reform, the equilibrium is Case j for any $(f_{12}, f_{21}) \in A_{ij}$. Note that there are many sets A_{ij} that are empty, in which case no exact equilibria exist, both pre- and post-reform. There are two regions A_{ij} where $i \neq j$ and $i, j \neq 0$: A_{24} and A_{13}. These are interesting because they represent the equilibria where a change in entry patterns following the policy reform. A_{13} means that after lowering the iceberg cost to export from 2 to 1, the entry pattern changed from Case 1, i.e. $(e_{12}, e_{21}) = (0, 0)$, to Case 3, i.e. $(e_{12}, e_{21}) = (1, 0)$, so the policy change induces country 2 to start exporting the differentiated good to 1, even though firms from 1 still do not export to 2. The A_{24} equilibrium is analogous in that the policy induces firms from 2 to start exporting, but in this case, both before and after the reform, firms from country 1 export to country 2. There are four nonempty sets A_{ii} where there is no change in the entry patterns after the reform, so we can expect the effect of the extensive margin to be minimal. The other non-empty sets A_{ij} have either $i = 0$ or $j = 0$; this means that before or after the reform, equilibria may fail to exist.

4.5 The Algorithm at Work: An Approximate Equilibrium

As before, suppose there are two countries $i = 1, 2$. Each has a measure $\theta_i = w_i L_i$ of firms that are considering which markets to enter. For simplicity consider only an environment with trade (i.e. no multinational production). An example allowing for multinational production is available upon request. International trade is subject to both iceberg and fixed costs, which can be asymmetric across country pairs. Consider a policy reform that lowers the iceberg costs for firms from 2 to export to 1 from $\tau_{12} = 2$ to $\tau'_{12} = 1$. Let $\tau_{21} = \tau'_{21} = \tau_{11} = \tau'_{11} = \tau_{22} = \tau'_{22} = 1$. The trade elasticity is $\sigma = 2$ and the expenditure share for the differentiated goods sector is $\mu = 1/2$. The other exogenous parameters are given below

$$w_1 = 1, \quad w_2 = 1, \quad L_1 = 1, \quad L_2 = 1, \quad \phi_1 = 2, \quad \phi_2 = 1$$

This implies

$$p_{11} = p'_{11} = 1, \quad p_{22} = p'_{22} = 2, \quad p_{12} = \frac{\sigma}{\sigma - 1} \frac{w_2 \tau_{12}}{\phi_2} = 4, \quad p_{21} = \frac{\sigma}{\sigma - 1} \frac{w_1 \tau_{21}}{\phi_1} = 1, \quad p'_{12} = \frac{\sigma}{\sigma - 1} \frac{w_2 \tau'_{12}}{\phi_2} = 2$$

Define

$$\begin{aligned}
Q_{11} &= \mu \frac{1}{\sigma} \left(\frac{p_{11}}{P_1} \right)^{1-\sigma} = \mu \frac{1}{\sigma} \left(\frac{P_1}{p_{11}} \right) = \frac{1}{4} \\
Q_{21} &= \mu \frac{1}{\sigma} \left(\frac{p_{21}}{P_2} \right)^{1-\sigma} = \mu \frac{1}{\sigma} \left(\frac{P_2}{p_{21}} \right) = \frac{1}{6} \\
Q_{22} &= \mu \frac{1}{\sigma} \left(\frac{p_{22}}{P_2} \right)^{1-\sigma} = \mu \frac{1}{\sigma} \left(\frac{P_2}{p_{22}} \right) = \frac{1}{12}
\end{aligned}$$

Then

$$\pi = \frac{\pi_{11} + \pi_{21} + \pi_{22} - 3\epsilon}{2} = \frac{\left(\frac{1}{4} + \frac{1}{6} + \frac{1}{12} \right)(1 + \pi) - 3\epsilon}{2} = \frac{1}{3} - 2\epsilon$$

Hence the restrictions are given by

$$\pi_{ij} > \epsilon \Leftrightarrow Q_{ij}(1+\pi) > \epsilon \Leftrightarrow Q_{ij}\left(\frac{4}{3} - 2\epsilon\right) > \epsilon \Leftrightarrow \epsilon < \frac{\frac{4}{3}Q_{ij}}{1+2Q_{ij}}$$

Applied to the three relevant cases, this yields the restrictions

$$\epsilon < \frac{\frac{1}{3}}{1+\frac{1}{2}} = \frac{2}{9}$$
$$\epsilon < \frac{\frac{2}{9}}{1+\frac{1}{3}} = \frac{1}{6}$$
$$\epsilon < \frac{\frac{1}{9}}{1+\frac{1}{6}} = \frac{2}{21}$$

Let $\epsilon = \frac{1}{20}$. Then we have $\pi = \frac{1}{3} - \frac{1}{10} = \frac{7}{30}$. Then the baseline parameterization is

$$f_{12} = \pi_{12} + \epsilon = \frac{61}{480}, \quad f_{11} = f_{22} = f_{21} = \epsilon = \frac{1}{20}$$

To show that there is no exact equilibrium after reform, note that either $e'_{12} = 1$ or $e'_{12} = 0$.

Consider first the case with $e'_{12} = 1$. We have $\pi' \in [\frac{1}{3} - \frac{8}{3}\frac{1}{20}, \frac{1}{3}]$, and $P_1 \in \{2, \frac{2}{3}\}$, depending on the entry decisions of other agents. If $P'_1 = 2$ and $e'_{11} = 0$, we must have $\pi'_{11} < f_{11}$ and $\pi'_{12} > f_{12}$. But $\pi'_{11} = \frac{1}{4}\frac{2}{1}(1+\pi') = \frac{1}{2}(1+\pi') \geq \frac{1}{2}\left(\frac{4}{3} - \frac{8}{3}\frac{1}{20}\right) = \frac{9}{15} > \frac{1}{20} = f_{11}$, a contradiction. If, on the other hand, $P'_1 = \frac{2}{3}$ and $e'_{11} = 1$, we must have $\pi'_{11} > f_{11}$ and $\pi'_{12} > f_{12}$. But in this case we have $\pi'_{12} = \frac{1}{4}\frac{\frac{2}{3}}{2}(1+\pi') = \frac{1}{12}(1+\pi') \leq \frac{1}{12}\frac{4}{3} = \frac{1}{9} < \frac{61}{480} = f_{12}$, a contradiction. Hence there is no exact equilibrium when $e'_{12} = 1$.

Then for exact equilibrium to exist, we must have $e'_{12} = 0$. In this case we need only consider the case when $P' = 1$ and $e'_{11} = 1$ because otherwise $P'_1 = 0$ (when $e'_{11} = 0$) and demand is not well-defined. If $e'_{12} = 0$, $P' = 1$, and $e'_{11} = 1$, we must have $\pi'_{11} > f_{11}$ and $\pi'_{12} < f_{12}$. But $\pi'_{12} = \frac{1}{4}\frac{1}{2}(1+\pi') \geq \frac{1}{8}\left(\frac{4}{3} - 2\frac{1}{20}\right) = \frac{1}{8}\frac{37}{30} = \frac{74}{480} > \frac{61}{480} = f_{12}$, a contradiction. Hence, exact equilibrium does not exist given this parameterization.

With no exact equilibrium, I now proceed to compute an approximate equilibrium. As in the algorithm, define the sets

$$\Omega_{i1} = \{j : \tau'_{ij} = \tau_{ij}, e_{ij} = 0\}, \quad \Omega_{i2} = \{j : \tau'_{ij} = \tau_{ij}, e_{ij} = 1\}$$
$$\Omega_{i3} = \{j : \tau'_{ij} < \tau_{ij}, e_{ij} = 0\}, \quad \Omega_{i4} = \{j : \tau'_{ij} < \tau_{ij}, e_{ij} = 1\}$$

There are $4N = 8$ sets. For this example these sets are

$$\Omega_{11} = \{\}, \quad \Omega_{12} = \{1\}, \quad \Omega_{13} = \{\}, \quad \Omega_{14} = \{2\}$$
$$\Omega_{21} = \{\}, \quad \Omega_{22} = \{1,2\}, \quad \Omega_{23} = \{\}, \quad \Omega_{24} = \{\}$$

Suppose the initial cutoffs are $k_{ij} = 100$, $\forall i = 1, 2$ and $j = 1, 2, 3, 4$. Given the cutoffs we have that the guess for the post-reform entry patterns are given by

$$E = \begin{bmatrix} e'_{11} & e'_{12} \\ e'_{21} & e'_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

This yields $P'_1 = \frac{2}{3} = P'_2$ and $\pi' = \frac{1}{3} - \frac{8}{3}\epsilon = \frac{1}{5}$. This in turn yields

$$\begin{aligned} \pi'_{11} &= \frac{1}{6}(1+\pi') = \frac{1}{5} > \frac{1}{20} = f_{11} \\ \pi'_{12} &= \frac{1}{12}(1+\pi') = \frac{1}{10} < \frac{61}{480} = f_{12} \\ \pi'_{21} &= \frac{1}{6}(1+\pi') = \frac{1}{5} > \frac{1}{20} = f_{21} \\ \pi'_{22} &= \frac{1}{12}(1+\pi') = \frac{1}{10} > \frac{1}{20} = f_{22} \end{aligned}$$

Hence the equilibrium is only 75% accurate.

There are no updates to the cutoffs of sets that are empty: $k^{(2)}_{11} = k^{(2)}_{13} = k^{(2)}_{21} = k^{(2)}_{23} = k^{(2)}_{24} = 100$ because $\hat{\Omega}^n_{11} = \hat{\Omega}^n_{13} = \hat{\Omega}^n_{21} = \hat{\Omega}^n_{23} = \hat{\Omega}^n_{24} = \{\}$. Further, $\hat{\Omega}^n_{12} = \{1\}, \hat{\Omega}^n_{14} = \{\}, \hat{\Omega}^n_{22} = \{1, 2\}$ yields $k^n_{12} = 100, k^n_{14} = 0, k^n_{22} = 100$. Let the Newton step be $\lambda = 0.2$. Then the guess for the next iteration is $k^{(2)}_{12} = \lambda k^n_{12} + (1-\lambda)k^{(1)}_{12} = 100, k^{(2)}_{14} = \lambda k^n_{14} + (1-\lambda)k^{(1)}_{14} = 80, k^{(2)}_{22} = 100$.

Because $k^{(2)}_{12} = 80$, we have

$$E^{(2)} = \begin{bmatrix} e'^{(2)}_{11} & e'^{(2)}_{12} \\ e'^{(2)}_{21} & e'^{(2)}_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

Which yields $P'^{(2)}_1 = 1,\ P'^{(2)}_2 = \frac{2}{3}$ and $\pi' = \frac{7}{30}$. This in turn yields

$$\begin{aligned} \pi'_{11} &= \frac{1}{4}(1+\pi') = \frac{37}{120} > \frac{1}{20} = f_{11} \\ \pi'_{12} &= \frac{1}{8}(1+\pi') = \frac{74}{480} > \frac{61}{480} = f_{12} \\ \pi'_{21} &= \frac{1}{6}(1+\pi') = \frac{37}{180} > \frac{1}{20} = f_{21} \\ \pi'_{22} &= \frac{1}{12}(1+\pi') = \frac{37}{360} > \frac{1}{20} = f_{22} \end{aligned}$$

As before, the equilibrium is only 75% accurate.

Likewise, there are no updates to the cutoffs of sets that are empty: $k^{(3)}_{11} = k^{(3)}_{13} = k^{(3)}_{21} = k^{(3)}_{23} = k^{(3)}_{24} = 100$ because $\hat{\Omega}^n_{11} = \hat{\Omega}^n_{13} = \hat{\Omega}^n_{21} = \hat{\Omega}^n_{23} = \hat{\Omega}^n_{24} = \{\}$. Further, $\hat{\Omega}^n_{12} = \{1\}, \hat{\Omega}^n_{14} = \{2\}, \hat{\Omega}^n_{22} = \{1, 2\}$ yields $k^n_{12} = 100, k^n_{14} = 100, k^n_{22} = 100$. Then the guess for the next iteration is $k^{(3)}_{12} = \lambda k^n_{12} + (1-\lambda)k^{(2)}_{12} = 100, k^{(3)}_{14} = \lambda k^n_{14} + (1-\lambda)k^{(2)}_{14} = 84, k^{(3)}_{22} = 100$.

Because $k_{12}^{(3)} = 84$, we have

$$E^{(3)} = \begin{bmatrix} e'^{(3)}_{11} & e'^{(3)}_{12} \\ e'^{(3)}_{21} & e'^{(3)}_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

Resulting in the same price indices and dividend per share as the second iteration: $P'^{(3)}_1 = 1$, $P'^{(3)}_2 = \frac{2}{3}$ and $\pi' = \frac{7}{30}$. Then as above we obtain $k_{12}^{(4)} = 100, k_{14}^{(4)} = \lambda k_{14}^n + (1-\lambda)k_{14}^{(3)} = 87.2, k_{22}^{(4)} = 100$.

Iterating in this fashion, we find $k_{14}^{(t)} = \lambda \cdot 100 + (1-\lambda)k_{14}^{(t-1)}$ which implies

$$\begin{aligned} & k_{14}^{(t)} - k_{14}^{(t-1)} = 0.2(100 - k_{14}^{(t)}) \\ & \Rightarrow k_{14}^{(t)} > k_{14}^{(t-1)}, \quad ||k_{14}^{(t+1)} - k_{14}^{(t)}|| < ||k_{14}^{(t)} - k_{14}^{(t-1)}|| \end{aligned}$$

So that $||k_{14}^{(t)} - k_{14}^{(t-1)}|| \to 0$ as $t \to \infty$. Suppose we set the tolerance to be 0.1. Then when does the iteration stop? We have

$$\begin{aligned} & k_{14}^{(5)} = \lambda k_{14}^n + (1-\lambda)k_{14}^{(4)} = 89.60 \\ & k_{14}^{(6)} = \lambda k_{14}^n + (1-\lambda)k_{14}^{(5)} = 91.81 \\ & k_{14}^{(7)} = \lambda k_{14}^n + (1-\lambda)k_{14}^{(6)} = 93.45 \\ & \dots \\ & k_{14}^{(19)} = \lambda k_{14}^n + (1-\lambda)k_{14}^{(18)} = 99.55 \\ & k_{14}^{(20)} = \lambda k_{14}^n + (1-\lambda)k_{14}^{(19)} = 99.64 \end{aligned}$$

And because $||k_{14}^{(20)} - k_{14}^{(19)}|| < 0.1$, the process stops and we settle on an approximate equilibrium that is 75% accurate, which in this case is the highest percentage possible given the number of available country pairs. It is easy to verify that the cutoffs for the other sets remain unchanged in the iterations leading up to algorithmic termination.