Self-Fulfilling Debt Restructuring

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Abstract

Sovereign defaults are costly not only because countries lose access to financial markets, but also because of output lost during the delays and haircuts associated with debt restructuring. Debt restructuring outcomes vary widely, and longer delays and larger haircuts are associated with higher spreads post-restructuring. I build a model with this novel mechanism of self-fulfilling debt restructuring in mind: higher spreads due to investor pessimism make it harder for countries to recover, thereby prolonging the restructuring process and lowering the amount of debt eventually recovered (raising the haircut), qualitatively replicating the aforementioned empirical regularities. Quantitatively, the model matches evidence on post-default restructuring for Argentina well, without impeding its ability to match standard pre-default business cycle statistics. Generating variation in post-default restructuring outcomes endogenously through variation in the spread and other pre-default aggregates points to the significant challenge of simultaneously matching both sets of moments successfully. Due to the inefficiencies associated with the restructuring process, there is scope for economic policies that mitigate rollover risk that gives rise to prolonged, self-fulfilling debt restructuring. To this end, I show that by lowering debt service requirements, longer debt maturity not only helps to circumvent default, but also shortens the duration of costly debt restructuring and raises the amount of debt recovered, highlighting a new channel by which maturity choice impacts efficiency and welfare.

Keywords: sovereign debt, debt crises, sovereign default, debt restructuring, haircuts, inefficient delay, debt maturity

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1 Introduction

Despite the prevalence of sovereign debt crises, their cause and resolution remain open questions that have relatively few definitive answers. One explanation for why countries default is because they face a sequence of negative output shocks and would prefer to default rather than repay and take a hit to its consumption. Central to this argument is the unenforceability of sovereign debt. This explanation relying on output risk is not sufficient as many debt crises feature countries growing positively (see e.g. Tomz and Wright, 2007) but still facing exorbitant interest rates. A complementary explanation is then that creditors coordinate on lending or no-lending equilibria and countries default not because they face adverse output shocks but because lenders chose not to lend so the sovereign is unable to roll over its debt. In this, I consider both these explanations and rather than study them as potential causes for debt crises, examine their importance for the resolution of such crises. If the causes of debt crises are not well-established, even less is known and understood about the resolution of such crises, especially those caused by liquidity, as opposed to insolvency, issues.

To address this issue, I build a model wherein lenders coordinating on a no-lending situation can not only trigger default, but also exacerbate the inefficiencies that arise during the resolution of the debt restructuring process. In the model, the sovereign trades a non-contingent bond with a continuum of risk-neutral investors. The sovereign has no commitment to repay its debt, and if it chooses to default, enters into a bargaining game with its creditors, who take collective action against it. The bargaining game ends in a settlement that the sovereign has to repay if it is to regain market access. The sovereign can choose to stay in exclusion if market conditions are severe enough that repaying the settlement would mean that its consumption would plummet in the period of reentry. There are two types of shocks. First, the sovereign is buffeted by standard output shocks that give rise to the incentive to default in order to smooth consumption over time. Second, a sunspot stipulates whether lenders coordinate on lending or not lending in that period and this lending behavior translates into liquidity shocks that trigger default when the sovereign is unable to roll over its debt. Similar to Aguiar et al. (2016), we assume that there is persistence in the coordination device, so that the lending situation today contains information about the lending situation tomorrow, and liquidity crises can continue for some time. These shocks not only affect the country's incentives to repay or default, but also affect its incentives to reenter markets and repay its settlement once it is already in default. This is because countries that face adverse lending conditions after having agreed on the settlement find it optimal to wait for lending conditions to improve in order to have a smoother consumption profile over time.

One key result is then that liquidity issues that arise from the willingness or unwillingness of creditors to lend can translate to bad outcomes not only pre-default, but post-default as well. The mechanism through which this arises is the following: adverse lending conditions lower the future value of regaining market access, thereby delaying market reentry, and by doing so, also shrink the size of the bargaining pie, thereby also reducing the value of the settlement. There is further feedback into the initial default decision as this will be the sovereign's value should it choose to default and restructure. The delays that arise from such self-fulfilling debt restructuring are inefficient because there is an output cost to staying in default that does not accrue to any party and is simply lost. This then opens to door for economic policies that mitigate the rollover risk that give rise to such self-fulfilling debt restructuring. I show that lengthening debt maturities can reduce the importance of rollover risk as longer debt maturity is isomorphic to less debt service and in the limit when debt service goes to zero, rollover risk does not matter. This then yields a novel welfare implication whereby longer debt maturity not only reduces the probability of default, but also reduces the inefficiencies that arise during debt restructuring should the country choose to default.

The paper makes four contributions. First, we establish that the standard model with only output risk accounts for only a small fraction of the heterogeneity in debt restructuring observed; as such, both output and rollover risks are necessary to generate the variation required. Haircuts are substantial (40 percent on average), and significantly different (standard deviation of 24 percent). At the same time, spreads also exhibit some variation (standard deviation of eight percent) that the same model must account for, in order to give an accurate representation of the dynamics surrounding a default episode. The standard model must have long-term debt and be parameterized in a particular way (Chatterjee and Eyigungor, 2009) to account for the variation in spreads among other important moments related to the default event (mean level of debt, mean level of spread); it is no surprise that the same model unchanged cannot match the same moments and the variations in the restructuring aggregates simultaneously. To match both sets of moments, the insight here is to use the spreads as a channel for generating variation in the restructuring variables, thereby hitting two birds with one stone. We achieve this by building a model wherein lenders' willingness to purchase debt impacts debt prices not only when the country is in good standing and has not defaulted, but also when the country is in bad standing and trying to regain market access. In the latter case, unwillingness to lend at affordable rates (or high enough prices) translates into worse restructuring outcomes (longer length in exclusion and higher haircuts) as the future value of regaining market access falls. Hence high spreads give rise to high haircuts, and low spreads give rise to lower haircuts, generating the variation required.

Second, we show how the self-fulfilling dynamics that arise from rollover risk have important implications not only for debt restructuring, but precisely because it affects restructuring outcomes, has an even more important role in the lead up to the default event as well, given the forward-looking nature of this class of models. The self-fulfilling dynamics that result from rollover risk have been shown to be an important component of recent debt crises (Cole and Kehoe 2000; Aguiar et al. 2016a); here we take the notion of self-fulfilling debt crises further by also allowing for self-fulfilling debt restructuring. Intuitively, if one believes that countries default because of liquidity (and not insolvency) reasons, then it stands to reason that they also restructure their debt taking into account future financing concerns. To assess the differential impact of rollover risk in the context of debt restructuring, we perform two decompositions. First we decompose the results and extract the impact of rollover risk in the environment with no restructuring. Then we compare this with the decomposition of results in the environment with restructuring. Quantitatively, we show that the impact of rollover risk is greater in the case with restructuring; this is encouraging in light of evidence that rollover risk may not be particularly important quantitatively for certain debt crises (Bocola and Dovis 2016). The intuition behind the result is that in being important also for post-default outcomes, the role of rollover risk is now amplified as its effects on pre-default outcomes are still present. Put differently, not only is lenders' pessimism bad because it induces default; pessimism is bad also because it also adversely impacts the subsequent restructuring process and impedes market access.

Third, the model's mechanism by which differences in spreads translate to different restructuring outcomes is validated by evidence documented in the empirical literature, allowing us to bridge the quantitative and empirical literatures in the process of testing the model's key implications. In the model, rollover risk affects restructuring as lenders' pessimism translates to longer lengths of capital maket exclusion because the borrower would prefer to wait for better lending conditions than take a hit to consumption today. Furthermore, pessimistic lenders lower the value of restructuring thereby reducing the bargaining surplus, which then lowers the amount of debt recovered and raises the haircut. Introducing rollover risk in the debt restructuring context then has two key testable implications: (1) high spreads due to rollover issues are accompanied by higher haircuts, and (2) high spreads due to rollover issues are associated with longer lengths in exclusion. Both these predictions are validated in the most comprehensive study of recent debt restructurings around the world done by Cruces and Trebesch (2013). Figure 1 shows that when one splits the restructuring episodes between 1970-2010 into two categories, those above the median haircut and those below median, that the high haircut group sees higher spreads post settlement, validating the first implication. There are two measures of exclusion: spreads exceeding a certain threshold or inability to issue new debt. By the first measure, we can also see from Figure 1 that the high haircut group (and hence high spread group) stayed longer in financial exclusion, consistent with the second implication. Alternatively, one can estimate a duration model as shown in Figure 2 and find that the probability of regaining market access is negatively correlated with the size of the haircut. Hence, introducing rollover risk in this context is not only a natural way to generate greater variation in the restructuring aggregates but can also be validated by its testable implications relating the spread to the haircut and the length of capital market exclusion.

Finally, we find that the welfare implications of introducing self-fulfilling expectations into a model with long-term debt restructuring are quantitatively significant. In the absence of rollover risk and debt restructuring, the standard model with long-term debt predicts lower welfare than the same model with short-term debt because debt dilution means agents borrow too much new long-term debt, raising default risk and lowering debt price.[1] The presence of rollover risk can reverse this result owing to the fact that long-term debt is a better instrument than short-term debt in terms of mitigating rollover issues. Rollover risk is modeled as alternating belief regimes that are persistent: under the high belief regime, lenders are optimistic and lend at better rates

[1] see Hatchondo, Martinez, and Sosa Padilla (2015) and Chatterjee and Eyigungor (2015)

than under the low belief regime, and belief regimes are persistent so that a high belief regime today implies that tomorrow's belief regime will also be high with positive probability. Holding fixed the belief regime, I find that the longer the maturity, the higher the consumption-equivalent welfare, indicating that the gains from mitigating rollover risk dominates the losses that come from debt dilution with longer maturity debt. Holding fixed debt maturity, we also find that the belief regime matters for welfare as well. In particular, the more time spent in the high-belief regime, the higher the consumption-equivalent measure of welfare, with more than ten percent difference between the regimes that yield the highest and lowest welfare levels. This is because in the model, long-term debt does not only help the sovereign circumvent default, it also reduces the inefficiencies that arise from costly delays during the restructuring process. These findings emphasize the nonnegligible impact of rollover risk in an environment that features sovereign debt, default, and restructuring.

The paper is related to several strands of literature. The two sources of risk considered imply that it is related both to the quantitative models of sovereign debt and default ala Arellano (2008) that model default in the tradition of Eaton and Gersovitz (1981),[2] and the models of self-fulfilling debt crises ala Cole and Kehoe (2000).[3] It builds directly on models of sovereign debt restructuring, e.g. Yue (2010) and Benjamin and Wright (2013), that heretofore abstract from self-fulfilling dynamics.[4] There are also related models that deal with debt restructuring in other contexts, e.g. Athreya et al. (2016) and Benjamin and Mateos-Planas (2014), and here the main difference lies in the fact that the insitutional features that matter (in sovereign debt there is no notion of bankrupcy, in contrast to consumer creidt) are fundamentally different hence the need to model things differently.[5] Finally, the paper is part of a growing literature the consider both fundamental and non-fundamental reasons for default where there is a potential for multiple equilibria.[6]

The rest of the paper proceeds as follows. The model environment and recursive formulation are presented in the next two sections. Section 4 presents a simplified model that highlights the key mechanism of the model and two qualitatively important testable implications. Section 5 presents event study examples that illustrate the dynamics implied by the model under specific shock realizations. The calibration is detailed in section 6, followed by the quantitative analysis in section 7. The last section concludes.

2 The Model Environment

Preferences and endowments. Consider a small open economy with identical households, whose

[2]see also Arellano and Ramanarayanan (2012), Hatchondo and Martinez (2009).

[3]see Aguiar et al. 2016. For a handbook treatment, see Aguiar and Amador 2014.

[4]see also Pitchford and Wright (2012), Schmitt-Grohe and Uribe (2016), Asonuma and Trebesch (2016), Bi (2008), and Chatterjee and Eyigungor (2015)

[5]see also Kovrijnykh and Szentes (2007) and Kovrijnykh and Livshits (2016)

[6]see Corsetti and Dedola (2016), Lorenzoni and Werning (2013), Cole and Kehoe (2000), Aguiar et al. (2016b), Calvo (1988), Conesa and Kehoe (2016)

preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where β denotes the discount factor, c_t denotes the consumption in period t and $u : \mathbb{R}_+ \to \mathbb{R}$ is the period utility function with standard properties: continuous, strictly increasing, strictly concave, and satisfies Inada conditions. There is a benevolent government that maximizes households' lfetime utility. In each period, households receive an endowment of a non-storable good y_t. The endowment y_t follows a first-order Markov process with support $Y = [\underline{y}, \bar{y}] \subset \mathbb{R}$ and transition function $f(y_t, y_{t+1})$.

Market structure and restructuring protocol. The government can borrow from or lend to international investors by issuing non-contingent bonds b' to a competitive pool of risk-neutral lenders. The government cannot commit to repay its debt. A country in good standing has the option to repay and issue new debt, or default and enter into negotiations to restructure its debt. Restructuring reduces the amount owed by a fraction that is the outcome of a bargaining game between the defaulting country and lenders that take collective action against it. While in default, the government suffers an output loss and has no access to financial markets. The sovereign gets the chance to reenter financial markets with probability θ, though the chance to reenter does not imply automatic reentry, as we allow the country stay in exclusion, implying that the length in exclusion depends not only on the parameter θ but also the value of repaying the restructured debt relative to the value of default.

Belief regimes, timing, and self-fulfilling dynamics. The value of repayment relies not only on the output shock and the amount of debt owed; in this environment with belief shocks, it also depends critically on the belief regime. Let the two-state Markov process ρ represent the belief of agents. If $\rho = \rho_B$, creditors coordinate on the rollover crisis equilibrium if the economy finds itself in the crisis zone. On the other hand, if $\rho = \rho_G$, the country faces no rollover issues in the current period but may face self-fulfilling crises in the future. Similar to Cole-Kehoe (2000), self-fulfilling crises can occur in this setup owing to the timing convention adopted: as the borrowing country can default in period, lenders' unwillingness to lend can trigger default as countries with sufficiently bad fundamentals (i.e. those in the crisis zone) require sufficient financing to stay in good standing. In contrast to Cole-Kehoe (2000), however, the belief regime matters regardless of whether the country is in good standing or bad standing: in good standing, bad beliefs $\rho = \rho_B$ can trigger default for sufficiently bad fundamentals that would not occur with $\rho = \rho_G$; in bad standing, bad beliefs $\rho = \rho_B$ can induce countries to choose default even when it receives the signal to reenter markets as the lack of investor financing implies it has to repay out of its current period income and take a hit to consumption. Finally, the belief regime also matters not only because of its in-period effects but also because the persistence induced by the Markov process for beliefs imply that current beliefs have implications for future beliefs and hence future values. Hence, even if the current belief regime is good $\rho = \rho_G$, this does not mean that self-fulfilling crises are ruled out completely (unless the belief process has an absorbing state), and the government takes this into account when deciding how much to issue and when to repay or default on its debt.

3 Recursive Formulation

I now define and characterize dynamic recursive equilibrium. The country's benevolent government seeks to maximize household's lifetime utility. It makes its asset position and default decisions based on the state vector $\mathbf{s} = (b, y, \rho)$, which consists of current asset position d, endowment shock y, and belief regime ρ.

If the country is in good standing, it has the option to repay or default on its debt:

$$v(d, y, \rho) = \max\{v_r(d, y, \rho), v_d(d, y, \rho)\}$$

If it chooses to repay, it solves the following problem

$$v_r(d, y, \rho) = \max_{c,d':c=y+q(d',y,\rho)[d'-(1-\lambda)d]-\lambda d} u(c) + \beta \int_{y,\rho} v(d', y', \rho') dg(\rho'|\rho) df(y'|y)$$

If it chooses to default instead, its value function is given by

$$v_d(y, \rho) = u(y - L(y)) + \beta\xi \int_{y,\rho} w(G(y', \rho'), y', \rho') dg(\rho'|\rho) df(y'|y) + \beta(1 - \xi) \int_{y,\rho} v_d(y', \rho') dg(\rho'|\rho) df(y'|y)$$

Note here that when the country gets the chance to reenter, it does not immediately go into good standing, but rather goes into a state of having settled its debt, with the option of repaying the settlement or staying in financial exclusion. And this value having already settled is denoted by $w(d, y, \rho)$, given by

$$w(d, y, \rho) = \max\{v_r(d, y, \rho), w_d(d, y, \rho)\}$$

The second component is the value of staying in exclusion $w_d(d, y, \rho)$ given by

$$w_d(d, y, \rho) = u(y - L(y)) + \beta\theta \int_{y,\rho} w(d, y', \rho') dg(\rho'|\rho) df(y'|y) + \beta(1 - \theta) \int_{y,\rho} w_d(d, y', \rho') dg(\rho'|\rho) df(y'|y)$$

Note that when the sovereign chooses to stay in exclusion post settlement, it is unable to issue new debt until it repays and regains market access.

To determine the settlement $G(y, \rho)$, one needs to form and then solve the bargaining problem. We will consider two different setups: standard Nash bargaining and flexible bargaining following Chatterjee-Eyigungor (2015). Under the Nash bargaining protocol with bargaining power α, restructured debt $\tilde{d}$ solves

$$\tilde{d} = \arg\max_{d} \left[\frac{v_d(d, y, \rho) - v_a(y)}{u'(y - L(y))} \right]^{\alpha} [q_b(d, y, \rho)d]^{1-\alpha}$$

where the borrower's outside option is its value in autarky

$$v_a(y, \rho) = u(y - L(y)) + \beta \int_y v_a(y')df(y'|y)$$

In the formulation above, the borrower's surplus is normalized by dividing through by the marginal utility so everything is denominated in terms of goods. One can alternatively just take the difference in values in which case the units are utils.

Due to computational issues (for certain regions in the parameter space, difficulties in convergence) associated with Nash bargaining, Chatterjee and Eyigungor (2015) introduce a notion of flexible bargaining that we adopt during the computation for the quantitative analysis. In this bargaining protocol, lenders make a take it or leave it offer to the sovereign whose outside option of the sovereign is augmented with a parameter α that mimics that bargaining power of the borrower in the Nash bargaining case. More specifically, in the period of default, the utility of the sovereign is given by $u(y - L(y))$ after which should the sovereign reject the offer, its welfare is given by the following augmented value in autarky

$$A(y) = u(y - \alpha) + \beta E_y A(y')$$

where α is the cost or benefit of being in autarky and allows the model to match the average debt recovery rate. Denote the level of debt that makes the sovereign indifferent between setting and going into autarky by $\bar{G}(y, \rho)$, then $\bar{G}(y, \rho)$ satisfies

$$E_{y,\rho} w(\bar{G}(y', \rho'), y', \rho') = E_y A(y')$$

When lenders make a take-it-or-leave-it offer $G(y, \rho)$, they take into account the borrower's participation constraint, with $G(y, \rho)$ given by

$$G(y, \rho) = \arg \max_{G \geq \bar{G}(y,\rho)} q_b(y, G, \rho)G$$

the settlement process internalizes the effect of pessimistic beliefs: these beliefs lower the borrower's settlement value for any given debt recovery rate, thereby reducing the set of feasible debt recovery rates. This leads to lower debt recovery rates and higher haircuts during periods of pessimism where spreads remain high regardless of output. Moreover, these high spreads post-restructuring associated with this coordination on the bad equilibrium also provides incentives for the sovereign to delay reentry until spreads falls in order to avoid the drop in consumption that comes with paying off the settlement. We will show in the next section that these effects are present not only with flexible bargaining, but also with Nash bargaining as well.

To complete the description of equilibrium, we need to characterize prices. To characterize prices, first define the following indicators: the default indicator

$$I(d,y,\rho) = \begin{cases} 1, & \text{if } v_r(d,y,\rho) < v_d(y,\rho). \\ 0, & \text{otherwise.} \end{cases}$$

and the exclusion indicator

$$J(d,y,\rho) = \begin{cases} 1, & \text{if } w_d(d,y,\rho) < v_r(d,y,\rho). \\ 0, & \text{otherwise.} \end{cases}$$

With these indicators in hand, we have that the price of new debt is given by

$$\begin{aligned} q(d',y,\rho) =& [1 - I(d,y,\rho)] \frac{E_{y,\rho}[1 - I(d',y',\rho')][\lambda + (1-\lambda)(z + q(D,y',\rho'))]}{1+r} \\ &+ [1 - I(d,y,\rho)] \frac{E_{y,\rho} I(d',y',\rho') \frac{P(y',\rho')}{d'})}{1+r} + I(d,y,\rho) \frac{P(y,\rho)}{d} \end{aligned}$$

The value of settlement $P(y,\rho)$ satisfies

$$P(y,\rho) = (1-\xi) E_{y,\rho} \frac{P(y',\rho')}{1+r} + \xi E_{y,\rho} \frac{q_b(G(y',\rho'),y',\rho') G(y',\rho')}{1+r}$$

The value then depends on

$$\begin{aligned} q_b(d,y,\rho) =& [1 - J(d,y,\rho)][\lambda + (1-\lambda)(z + q(D(d,y,\rho),y,\rho))] + \\ & J(d,y,\rho) \frac{[(1-\theta) E_{y,\rho} q_b(d,y',\rho')}{1+r} + J(d,y,\rho) \frac{\theta E_{y,\rho} J(d,y',\rho') q_b(d,y',\rho')}{1+r} + \\ & J(d,y,\rho) \frac{\theta E_{y,\rho}[1 - J(d,y',\rho')][\lambda + (1-\lambda)(z + q(D,y',\rho'))]]}{1+r} \end{aligned}$$

4 Simplified Environment: Understanding the Main Mechanism

To better illustrate the mechanism and make clear the role of the different modeling elements and assumptions, we now specialize the environment to one with no output risk ($y_t = y, \forall t$) and no exogenous sources of delay ($\theta = 1$, $\xi = 1$). We consider three simplified versions of the model: first a version with short-term debt $\delta = 1$ and an exogenous haircut rule G, one with short-term debt where the haircut is endogenously determined through standard Nash bargaining, and finally a version with long-term debt and endogenous Nash bargaining. The objective is to show first how rollover risk creates inefficient delay, followed by differences in both the time to reentry and debt recovery rate, before finally showing how maturity choice can be used not only to circumvent default, but also facilitate earlier reentry and greater debt recovery.

4.1 Exogenous Haircuts, Endogenous Delays

In addition to the assumptions on output risk, debt maturity and exogenous sources of delay highlighted above, we will also start by assuming the belief regimes are permanent (i.e. perfectly persistent), the default cost is linear $L(y) = \kappa y$, risk-free interest rate r is such that the risk-free price of debt is β, and settlement follows a rule $G(\rho) = G$. For this subsection, we will assume G is given. With this set of assumptions, the recursive formulation for the general problem reduces to the following: in good standing, the value function for the government with the option to repay or default is given by

$$v(d, \rho) = \max\{v_r(d, \rho), v_d(d, \rho)\}$$

If it chooses to repay, it solves the following problem

$$v_r(d, \rho) = \max_{d'} u(y + q(d', \rho)d' - d) + \beta v(d', \rho)$$

By contrast, the value in default $v_d(G, \rho)$ is given by

$$v_d(G, \rho) = u(y - \kappa y) + \beta w(G, \rho)$$

where the value of settlement $w(d, \rho)$ is

$$w(d, \rho) = \max\{v_r(d, \rho), w_d(d, \rho)\}$$

and the value of staying in exclusion $w_d(d, \rho)$ is

$$w_d(d, \rho) = u(y - \kappa y) + \beta w(d, \rho)$$

The pricing functions are also simplified due to the aforementioned assumptions, and we skip them for the sake of brevity.

We first show that given this predetermined settlement rule, both the decision to reenter following default and the decision to default decline monotonically with the belief regime. Hence, if the country decides to reenter when creditors are optimistic, then they will reenter when creditors are pessimistic, but not the other way around; similarly for the decision to default. This means then that because investor pessimism inhibits reentry, it creates inefficient delay.

Proposition 1. *Fix the settlement rule $G(\rho) = G$. Given income level y and default cost $\kappa = 0.5y$, we can show that*
(i) Exclusion $J(d, \rho)$ is decreasing in ρ

(ii) Default $I(d, \rho)$ is decreasing in ρ
Rollover risk creates inefficient delay.

Proof. Suppose the output cost of default is $\kappa = 0.5y$.. What is optimal exclusion policy after recovery? In the case when you reenter your continuation value is

$$v_r(G, y) = \frac{u\left(y - (1-\beta)G\right)}{1-\beta}$$

This continuation value is clearly decreasing in G, so does there exist a G such that it is better to just stay in default? Staying in default would imply the continuation value

$$w_d(G, y) = \frac{u\left(0.5y\right)}{1-\beta}$$

The equilibrium $w(G, y)$ that solves

$$w(G, \rho) = \{v_r(G, \rho), w_d(G, \rho)\}$$

then has the standard property of staying in default if the haircut is too large. What is too large? Given v_r is decreasing in G while w_d is independent of G, there exists a $\bar{G}$ that solves

$$\frac{u\left(y - (1-\beta)\bar{G}_H\right)}{1-\beta} = \frac{u\left(0.5y\right)}{1-\beta}$$

With this threshold condition, the exclusion policy is given by

$$J(G, \rho_H) = \begin{cases} 1, & \text{if } G > \bar{G}_H. \\ 0, & \text{otherwise.} \end{cases}$$

The condition that determines $\bar{D}_H$ must reflect what happens after default, in particular, given $\bar{G}_H$

$$\frac{u\left(y - (1-\beta)\bar{D}_H\right)}{1-\beta} = \begin{cases} \frac{u(0.5y)}{1-\beta}, & \text{if } G > \bar{G}_H. \\ u(0.5y) + \frac{\beta u(y-(1-\beta)G)}{1-\beta}, & \text{otherwise.} \end{cases}$$

Thus far, the characterization has been for the high belief regime where there is effectively no rollover risk. Suppose now the government is always subject to a run every period (the low belief regime prevails all the time). What happens? In terms of the continuation value, for small enough G (i.e. $G < \bar{G}_L$), the price is still β as before; but for the cases where funding is necessary to rollover, the price goes to zero as the debt is never repaid. The safe zone $G < \bar{G}_L$ for the continuation is given by

$$\frac{u\left(y - \bar{G}_L\right)}{1-\beta} = \frac{u(0.5y)}{1-\beta}$$

Note that given the similarities in the RHS of the two conditions for $\bar{G}_L$ and $\bar{G}_H$, the different

LHS imply that $\bar{G}_L = (1-\beta)\bar{G}_H < \bar{G}_H$ so an exclusion zone indeed exists. The exclusion policy function under the low belief regime is then

$$J(G, \rho_L) = \begin{cases} 1, & \text{if } G > \bar{G}_L. \\ 0, & \text{otherwise.} \end{cases}$$

Hence we have that $J(G, \rho_H) \leq J(G, \rho_L)$ with strict inequality when $\bar{G}_L \leq G < \bar{G}_H$, as desired.

Now consider the effect of this on initial default decision, where the relevant threshold $\bar{D}_L$ satisfies the condition

$$\frac{u\left(y - \bar{D}_L\right)}{1-\beta} = \begin{cases} \frac{u(0.5y)}{1-\beta}, & \text{if } G \geq \bar{G}_L. \\ u(0.5y) + \frac{\beta u(y-(1-\beta)G)}{1-\beta}, & \text{otherwise.} \end{cases}$$

To see how $\bar{D}_H$ compares with $\bar{D}_L$, note that we can write

$$\frac{u\left(y - \bar{D}_L\right)}{1-\beta} = \begin{cases} \frac{u(0.5y)}{1-\beta}, & \text{if } G \geq \bar{G}_H. \\ u(0.5y) + \frac{\beta u(0.5y)}{1-\beta}, & \text{if } \bar{G} > G \geq \bar{G}_L. \\ u(0.5y) + \frac{\beta u(y-(1-\beta)G)}{1-\beta}, & \text{otherwise.} \end{cases}$$

By contrast remember

$$\frac{u\left(y - (1-\beta)\bar{D}_H\right)}{1-\beta} = \begin{cases} \frac{u(0.5y)}{1-\beta}, & \text{if } G \geq \bar{G}_H. \\ u(0.5y) + \frac{\beta u(y-(1-\beta)G)}{1-\beta}, & \text{if } \bar{G}_H > G \geq \bar{G}_L. \\ u(0.5y) + \frac{\beta u(y-(1-\beta)G)}{1-\beta}, & \text{otherwise.} \end{cases}$$

So we know outside the crisis zone we have $\bar{D}_L = (1-\beta)\bar{D}_H < \bar{D}_H$. To compare the two in the crisis zone, note that there is a jump at $\bar{G}_L$ in $\bar{D}_L$'s condition while the RHS is continuous in the condition for $\bar{D}_H$. At $\bar{G}_L$, we want to show that as $\bar{D}_L$ falls to $\bar{G}_L = 0.5y$, it is smaller than $\bar{D}_H$. This is not hard to see: at $G = \bar{G}_L$,

$$\begin{aligned} \frac{u\left(y - (1-\beta)\bar{D}_H\right)}{1-\beta} &= u(0.5y) + \frac{\beta u(y-(1-\beta)G)}{1-\beta} \\ &= u(y - \bar{D}_L) + \frac{\beta u(y-(1-\beta)\bar{G}_L)}{1-\beta} \\ &= u(y - \bar{G}_L) + \frac{\beta u(y-(1-\beta)\bar{G}_L)}{1-\beta} \end{aligned}$$

By concavity, the last equality implies $(1-\beta)\bar{D}_L = (1-\beta)\bar{G}_L < (1-\beta)\bar{D}_H < \bar{G}_L$, and the first of these inequalities implies $\bar{D}_L < \bar{D}_H$ as desired.

Furthermore, as G goes from $\bar{G}_L$ to $\bar{G}_H$, $\bar{D}_L$ stays the same, while $\bar{D}_H$ increases as G increases (the RHS goes down as G goes up, and as the RHS goes down, for the equality to continue to hold, $\bar{D}_H$ has to go up). Hence we still have $\bar{D}_L < \bar{D}_H$ as desired.

To complete the proof, we then have given the definition of the indicator functions

$$I(D, \rho_L) = \begin{cases} 1, & \text{if } D > \bar{D}_L. \\ 0, & \text{otherwise.} \end{cases}$$

and

$$I(D, \rho_H) = \begin{cases} 1, & \text{if } D > \bar{D}_H. \\ 0, & \text{otherwise.} \end{cases}$$

that $\bar{D}_L \leq \bar{D}_H$ with strict inequality when $\bar{D}_L \leq D < \bar{D}_H$ that $I(D, \rho_L) \geq I(D, \rho_H)$, with strict inequality when $\bar{D}_L \leq D < \bar{D}_H$, as deisred.

□

To understand the result, it is useful to work backwards. Suppose the sovereign has already defaulted and bargained with its creditors. Then given that belief regimes are permanent and there is no output risk, the government either reenters immediately or chooses to stay in default forever. This then simplifies the subsequent analysis: given that the value of staying in default does not depend on the settlement while the value of reentering decreases with the settlement, we know that the reentry decision can be characterized by a cutoff $\bar{G}$. This cutoff will vary with the belief regime as the cutoff with pessimistic investors is the maximum amount that the government can sustain without financing, while the cutoff with optimistic investors takes into account market financing. Then given that more debt can be sustained with financing, there exists a region - the exclusion zone - where the sovereign only reenters with optimistic investors, hence inefficient delay with greater rollover risk as manifested in pessimism among investors. This is illustrated in the next figure.

Given the thresholds for reentry and the settlement rule, we can then also characterize the initial default decision. The default decision is still characterized by a cutoff rule because the value of default still does not depend on the initial debt level (though the reason differs from that above) while the value of repayment decreases with the debt level. The value of default does not depend on the initial debt level because the settlement rule does not depend on the initial debt level. Hence we have cutoffs $\bar{D}$ for initial default that also differ across belief regimes. Once again a zone exists where the sovereign only repays if investors are optimistic and defaults if investors are pessimistic - the crisis zone. While the cutoffs that define the exclusion zone do not depend on the crisis zone, the crisis zone thresholds can depend on the thresholds of the exclusion zone. To understand this, note that if the settlement rule stipulates a settlement value G that lies in the exclusion zone, this means the sovereign chooses to settle and reenter immediately with optimistic investors, and this means that the continuation value from defaulting in the first period is then the value of reentering with the stipulated settlement value G, whereas the sovereign chooses to stay in exclusion under the low belief regime, implying that the continuation value of defaulting after restructuring equals the value of default, which is tied to the loiwer boundary of exclusion zone, as this cutoff is equal to the output loss in default. More succinctly, we have that if $\kappa y = \bar{G}_L < G < \bar{G}_H = \kappa y/(1-\beta)$ then

$$\bar{D}_L = \kappa y = \bar{G}_L$$

while

$$G < \bar{D}_H < \kappa y/(1-\beta) = \bar{G}_H$$

And one can proceed similarly with the other cases. In general we know that $\bar{D}_L \leq \bar{G}_L$ and $\bar{D}_H \leq \bar{G}_H$.

4.2 Endogenous Haircuts, Endogenous Delays

Now we consider a slightly richer model where settlement is endogenous and proceeds through Nash bargaining. We use standard Nash bargaining in this case to show that the forces at play do not require the flexible bargaining assumption we adopt in the quantitative anaylsis. (The proof below still goes through even if one stipulates flexible bargaining.) In this subsection, the objective is twofold: first to show that the earlier result stipulating that rollover risk creates inefficient delay even when settlement is endogenous, and second, that the settlement that arises from the bargaining process has the property that larger haircuts (lower debt recovery rates) are associated with the higher spreads that come with greater rollover risk.

Proposition 2. *Given income level y and default cost $\kappa = 0.5y$, we can show that*
(i) Exclusion $J(d,\rho)$ is decreasing in ρ
(ii) Settlement $G(\rho)$ is increasing in ρ
Rollover risk creates inefficient delay and lowers debt recovery.

Proof. The proof for $J(d,\rho)$ is decreasing in ρ follows the proof given earlier, as we still have the thresholds $\bar{G}_H$ and $\bar{G}_L$ satisfying

$$\frac{u\left(y-(1-\beta)\bar{G}_H\right)}{1-\beta} = \frac{u\,(0.5y)}{1-\beta}$$

and

$$\frac{u\left(y-\bar{G}_L\right)}{1-\beta} = \frac{u(0.5y)}{1-\beta}$$

Note that given the similarities in the RHS of the two conditions for $\bar{G}_L$ and $\bar{G}_H$, the different LHS imply that $\bar{G}_L = (1-\beta)\bar{G}_H < \bar{G}_H$ so an exclusion zone indeed exists. Given the threshold condition, the exclusion policy under the high belief regime is given by

$$J(G,\rho_H) = \begin{cases} 1, & \text{if } G > \bar{G}_H. \\ 0, & \text{otherwise.} \end{cases}$$

and the exclusion policy under the low belief regime is

$$J(G,\rho_L) = \begin{cases} 1, & \text{if } G > \bar{G}_L. \\ 0, & \text{otherwise.} \end{cases}$$

Hence we have that $J(G,\rho_H) \leq J(G,\rho_L)$ with strict inequality when $\bar{G}_L \leq G < \bar{G}_H$, as desired.

Now consider the determination of $G^*(\rho)$. We know that given the environment there are only three equilibrium paths in this case: (i) the country never defaults, (ii) the country defaults in period 1 and reenters immediately, and (iii) the country defaults in period 1 and stays in default forever.

If the country never defaults in all states of the world (suppose default cost is simply too high), $G^*(\rho)$ is degenerate and immaterial and the claim is trivially satisfied. Similarly if the parameterization is such that the country defaults in both regimes and prefers to stay in exclusion forever, $G^*(\rho)$ is degenerate and immaterial and the claim is trivially satisfied.

Hence consider the interesting cases where the country finds it optimal to default and reenter with positive probability. Compute the G^* that solves the Nash bargaining problem assuming immediate reentry following restructuring. The class of Nash bargaining problems we consider admits a unique solution. if $G^* < \bar{G}_L$, this settlement will prevail in both belief regimes and we have $G(\rho_H) = G(\rho_L) = G^*$. However, if $G^* \in (\bar{G}_L, \bar{G}_H)$, then this settlement will not prevail in the low regime, and will only do so in the high belief regime. (Whether this is true depends on the parameterization of y, κ, β). In this case, one has to solve the constrained bargaining problem that stipulates repayment only if $G \leq \bar{G}_L$, and solve for the settlement given this additional constraint. This implies that in equilibrium $G(\rho_L) \leq \bar{G}_L$, while no such constraint applies to $G(\rho_H)$. Hence $G(\rho_L) \leq G(\rho_H)$ as desired.

□

To understand this proposition, first note that the part of the proof concerning the reentry decision does not depend on the bargaining protocol. This is because there is no incentive for a country to restructure multiple times: all uncertainty is resolved after the first period, and the equilibrium settlement does not depend on the initial debt held by the sovereign just before defaulting. Put differently, if after the initial settlement the equilibrium settlement is $G^*(\rho)$ and the country chooses to stay in exclusion (i.e. default again), the restructuring outcome would still stipulate $G^*(\rho)$ and hence the country continues to stay in exclusion and so forth. Key to this result is the property that in this class of bargaining problems, bygones are bygones: the debt the sovereign carries into the bargaining problem does not matter for the bargaining outcome.

What does matter for the bargaining outcome is the future value post-restructuring, and this is why the self-fulfilling debt restructuring mechanism we emphasize here has an impact on equilibrium settlement. In particular, because future values differ, the cutoffs that characterized the reentry decision differ, and as we've characterized and explained before, the sovereign may reenter when investors are optimistic but default even investors are pessimistic because $\bar{G}_L < \bar{G}_H$. This condition will be important for determining the equilibrium settlement. The cases where the country never defaults or stays in default forever are uninteresting from the standpoint of the determining an endogenous, nondegenerate settlement, so we focus on the case of initial default, bargaining, settlement and reentry. Given this reentry pattern we can solve the unconstrained Nash bargaining problem, which will have unique solution G^*. If this unconstrained solution is low enough, i.e. $G < \bar{G}_L < \bar{G}_H$ so neither constraint binds, then the equilibrium settlement is the same across belief regimes. However, if either $\bar{G}_L < G < \bar{G}_H$ or $\bar{G}_L < \bar{G}_H < G^*$ then we know that the equilibrium settlement with pessimistic investors $G(\rho_L)$ must be lower than that with optimistic investors $G(\rho_H)$ as these objects have to satisfy the constraints $G(\rho_H) \leq \bar{G}_H$ and $G(\rho_H) \leq \bar{G}_L$ and we know $\bar{G}_L < \bar{G}_H$. Intuitively, as the future value is higher when investors are optimistic, the value to restructuring is greater in the high belief regime as the bargaining pie is bigger, and this in turn leads to higher debt recovery rate $G(\rho)$. We will discuss this in greater detail below.

4.3 Debt Maturity and Self-Fulfilling Restructuring

Default due to rolloer risk can effectively be mitigated by lower debt service: in the limiting case when debt service goes to zero, whether investers coordinate on a lending or no-lending situation has no impact on equilibrium. Debt service is lower and longer the maturity of debt: here we appeal to the standard assumption of not distinguishing between long-term debt issued today and long-term debt issued tomorrow so that the maturity parameter δ controls both the amount of debt that comes due and must be serviced in each period as well as the expected duration of the instrument. The next proposition shows how maturity choice can be used to mitigate the effect of rollver risk on both pre- and post-default (restructuring) equilibrium objects: When one fixes the amount of initial debt, there exists a sufficiently long instrument that can rule out default. Conversely, if one fixes debt maturity to be either δ_1 or δ_2, there will be debt levels high enough to trigger default and restructuring. In this case, we will show that the case with longer term debt (δ_1 if $\delta_1 < \delta_2$) will be associated with greater debt recovery and less delays in restructuring.

Proposition 3. *Given income level y and default cost $\kappa = 0.5y$, we can show that*
(i) Fix debt level d. $\exists \delta^*$ such that the country does not default in either belief regime

(ii) Consider two different maturity parameters $\delta_1 < \delta_2$. Suppose d is sufficiently large to induce default. Then $J(d, \rho; \delta_1) < J(d, \rho; \delta_2)$.
(iii) Consider two different maturity parameters $\delta_1 < \delta_2$. Suppose d is sufficiently large to induce

default. Then $G(d,\rho;\delta_1) > G(d,\rho;\delta_2)$.
LT debt mitigates rollover risk by not only lowering the probability of default, but also lowering the haircut, and reducing the length of costly delay.

Proof. Part(i): To ensure no default in either belief regime, we need to pick $\delta^* \leq \bar{\delta}$, where $\bar{\delta}$ satisfies

$$\frac{u\left(y-\bar{\delta}d\right)}{1-\beta} = \frac{u(0.5y)}{1-\beta}$$

Part(ii) We know d is high enough to induce default in both cases. Define $\bar{G}(\rho_H;\delta_1) = \bar{G}_{H,1}$ and $\bar{G}(\rho_L;\delta_1) = \bar{G}_{L,1}$ that solve

$$\frac{u\left(y-\delta_1\bar{G}_{H,1}+q\left[\bar{G}_{H,1}-(1-\delta_1)\bar{G}_{H,1}\right]\right)}{1-\beta} = \frac{u\left(y-\delta_1[1-q]\bar{G}_{H,1}\right)}{1-\beta} = \frac{u\left(0.5y\right)}{1-\beta}$$

where q satisfies $q = \beta[\delta_1 + (1-\delta_1)q] \Rightarrow q = \frac{\beta\delta_1}{1-\beta(1-\delta_1)} \Rightarrow 1-q = \frac{1-\beta}{1-\beta(1-\delta_1)}$ and

$$\frac{u\left(y-\delta_1\bar{G}_{L,1}\right)}{1-\beta} = \frac{u(0.5y)}{1-\beta}$$

Similarly define $\bar{G}_{H,2}$ and $\bar{G}_{L,2}$ to satisfy

$$\frac{u\left(y-\delta_2\bar{G}_{H,1}+\delta_2(1-q)\bar{G}_{H,2}\right)}{1-\beta} = \frac{u\left(0.5y\right)}{1-\beta}$$

and

$$\frac{u\left(y-\delta_2\bar{G}_{L,2}\right)}{1-\beta} = \frac{u(0.5y)}{1-\beta}$$

Note that

$$\frac{dq}{d\delta} = \frac{[1-\beta(1-\delta)]-\beta\delta}{1-\beta(1-\delta)} = \frac{\beta(1-\beta)}{1-\beta(1-\delta_1)}$$

Hence

$$\frac{d\delta(1-q)}{d\delta} = \frac{1-\beta}{1-\beta(1-\delta_1)} - \frac{(1-\beta)\beta\delta}{[1-\beta(1-\delta_1)]^2} = \frac{(1-\beta)^2}{[1-\beta(1-\delta_1)]^2} > 0$$

So that because $\delta_1 < \delta_2$ we have that $\bar{G}_{L,1} > \bar{G}_{L,2}$ and $\bar{G}_{H,1} > \bar{G}_{H,2}$. Hence we have $J(d,\rho_H;\delta_1) > J(d,\rho_H;\delta_2)$ and $J(d,\rho_L;\delta_1) > J(d,\rho_L;\delta_2)$.
Part (iii): Now suppose the LT bonds have coupons such that in equilibrium that bond price is $q=\beta$ for both δ_1 and δ_2. This ensures that the optimal debt issuance policy following the restructuring is to keep issue the amount of the settlement $B_{t+1} = G(\rho;\delta), \forall t$ (otherwise consumption is not smooth which cannot be optimal given the equality of the discount factor with the shadow value implied by the debt price, or the transversality condition is violated). With this in hand, we have that the borrower's surplus is given by

$$\Delta_b(G;\delta,\rho) = u(y-\kappa y) + \beta\frac{u(y-\delta(1-q)G)}{1-\beta} - \frac{u(y-\kappa y)}{1-\beta}, \quad \text{if } G \leq \bar{G}(\rho;\delta)$$

While the lender's surplus

$$\Delta_l(G;\delta,\rho) = q(G;\delta,\rho)G$$

The objects of interest solve

$$\begin{aligned}
G^*_{\delta_1,\rho} &= \arg \max_{G\leq \bar{G}(\rho;\delta_1)} \Delta_b(G;\delta_1,\rho)^{\alpha}\Delta_l(G;\delta_1,\rho)^{1-\alpha} \\
G^*_{\delta_2,\rho} &= \arg \max_{G\leq \bar{G}(\rho;\delta_2)} \Delta_b(G;\delta_2,\rho)^{\alpha}\Delta_l(G;\delta_2,\rho)^{1-\alpha}
\end{aligned}$$

If the solution is interior, it satisfies the optimality condition given by

$$\frac{\alpha}{1-\alpha} = \frac{\beta\delta(1-q)u'(y-\delta(1-q)G)}{q(1-\beta)} = u'(y-\delta(1-q)G)$$

So if the constraint doesn't bind in either case, $\delta_1 < \delta_2$ and the optimality condition above would imply

$$u'(y-\delta_1(1-q_1)G) = u'(y-\delta_2(1-q_2)G), \quad q_i = \frac{\beta\delta_i}{1-\beta(1-\delta_i)} \Rightarrow \delta_1(1-q_1) < \delta_2(1-q_2)$$

Denote the unconstained (interior) solution to the two bargaining problems above by $\tilde{G}^*_{\delta_1}$ and $\tilde{G}^*_{\delta_2}$, respectively. Suppose for contradiction that $\tilde{G}^*_{\delta_1} \leq \tilde{G}^*_{\delta_2}$ then

$$\begin{aligned}
\delta_1(1-q_1)\tilde{G}^*_{\delta_1,\rho} < \delta_2(1-q_2)\tilde{G}^*_{\delta_2,\rho} &\Rightarrow u'(y-\delta_1(1-q_1)\tilde{G}^*_{\delta_1,\rho}) < u'(y-\delta_2(1-q_2)\tilde{G}^*_{\delta_2,\rho}) \\
&\Rightarrow \frac{\alpha}{1-\alpha} < \frac{\alpha}{1-\alpha}
\end{aligned}$$

a contradiction, hence we must have $\tilde{G}^*_{\delta_1} > \tilde{G}^*_{\delta_2}$. If the constraints on the bargaining problem do not bind, then we have $G^*_{\delta_1} = \tilde{G}^*_{\delta_1} > \tilde{G}^*_{\delta_2} = G^*_{\delta_2}$ as desired. Now consider the constraints: we know from the proof of Part (ii) that $\bar{G}(\rho;\delta_1) > \bar{G}(\rho;\delta_2)$. Given that (1) the unconstrained solutions $\tilde{G}^*_{\delta_1}$ and $\tilde{G}^*_{\delta_2}$ satisfy $\tilde{G}^*_{\delta_1} > \tilde{G}^*_{\delta_2}$ and (2) the thresholds satisfy $\bar{G}(\rho;\delta_1) > \bar{G}(\rho;\delta_2)$, the equilibrium debt recovery rates $G^*_{\delta_1}$ and $G^*_{\delta_2}$ must then satisfy

$$G^*_{\delta_2} = \min\{\tilde{G}^*_{\delta_2}, \bar{G}(\rho;\delta_2)\} < \min\{\tilde{G}^*_{\delta_1}, \bar{G}(\rho;\delta_1)\} = G^*_{\delta_1}$$

as desired. □

The first part of the proposition precluding default is intuitive: by lowering debt service past the output cost of default, one creates sufficient incentive for the sovereign never to default. Lowering debt service as explained is tantamount to lengthening debt maturity. The arguments regarding the impact of long-term debt on the restructuring aggregates are more involved. The intuition regarding the monotonicity in the reentry decision for different maturities differs from the intuition for different belief regimes that we discussed earlier. Earlier the intuition regarding the differences in the reentry decision boiled down to the availability of financing; in this case, there are two effects present - an effect on debt service and a price effect - and as the proof above shows, these effects both combine to ensure that if the country enters with long-term debt, it does so with short-term debt as well, but not necessarily the other way around. Intuitively with longer maturities, because debt service is lower today, and the optimal bond policy is such that it remains lower tomorrow and in the future, this lowers default risk and hence provides incentives for the sovereign to repay even when the debt is higher. Hence longer maturities reduce the length of costly delays. Turning now to the debt recovery rate, the proof shows us that there are two key components that determine the settlement: if the solution is interior and unconstrained, then the ratio of the relative surpluses have to be equal to the ratio of bargaining powers, and if it is constrained, then it depends also on the thresholds determining reentry that we've just discussed. On both counts, longer maturities imply larger settlements and higher debt recovery rates. To understand this, first consider the unconstrained solution. As the ratio of bargaining powers is the same across maturity levels, the ratio of marginal surpluses also has to be the same. However, if the settlement was the same or smaller in the case with longer debt maturity, combined with lower debt service that means the marginal utility would be higher with long-term debt, in contradiction to the equality of the ratio of marginal surpluses with the ratio of bargaining powers. Hence, the settlement the solves the unconstrained problem has to be larger for the case with longer maturity. Furthermore, including the constraints only reinforces the result, as the cutoffs for the settlement is higher in the case with long-term debt, so that even accounting for constraints, longer-term debt results in larger settlements and higher debt recovery rates.

4.4 Self-Fulfilling Restructuring and Welfare

Given the output cost incurred while in default, delays in reentry following a restructuring are costly and inefficient. We have just seen how longer maturities can not only help countries circumvent default but also reduce the inefficient delays associated with restructuring. These two effects serve to increase the sovereign's welfare and improve efficiency. Longer maturities also imply higher debt recovery rates, and that may seem to imply lower welfare for the sovereign, who carries more debt out of restructuring; but this need not be the case given that the sovereign only repays a fraction of the settlement every period, and the price also reflects the different payout stream implied by the longer instrument. The next proposition makes clear the welfare implications of debt maturity choice in our environment with self-fulfilling debt restructuring.

Proposition 4. *Given income level y and default cost $\kappa = 0.5y$, welfare with $\delta_2 = \delta < 1$ (LT) is higher than welfare with $\delta_1 = 1$ (ST), and even more so with restructuring. In particular, the welfare gains in going from the low belief regime to the high belief regime and using LT debt as* *opposed to ST debt is given by*

$$V_\delta(D,\rho_H)-V(D,\rho_L)=\begin{cases} u(0.5y)+\frac{\beta u(\frac{1-\alpha}{\alpha})}{1-\beta}-\frac{u(0.5y)}{1-\beta}, & \bar{D}_{H,\delta}\leq D \\ \frac{u(y-\delta[1-q]D)}{1-\beta}-u(0.5y)-\frac{\beta u(\frac{1-\alpha}{\alpha})}{1-\beta}+u(0.5y)+\frac{\beta u(\frac{1-\alpha}{\alpha})}{1-\beta}-\frac{u(0.5y)}{1-\beta} & \bar{D}_{L,\delta}\leq D\leq \bar{D}_{H,\delta} \\ \frac{u(y-\delta[1-q]D)}{1-\beta}-\frac{u(0.5y)}{1-\beta} & \bar{D}_L\leq D\leq \bar{D}_{L,\delta} \\ \frac{u(y-\delta[1-q]D)}{1-\beta}-\frac{u(y-(1-\beta)D)}{1-\beta} & D\leq \bar{D}_L \end{cases}$$

Proof. Recall that for ST debt you have

$$\frac{u\left(y-\bar{G}_L\right)}{1-\beta}=\frac{u(0.5y)}{1-\beta}$$

and

$$\frac{u\left(y-(1-\beta)\bar{G}_H\right)}{1-\beta}=\frac{u\left(0.5y\right)}{1-\beta}$$

Denote the solution to the bargaining problem by $G^*(\rho_H)$ and $G^*(\rho_L)$ then the thresholds for default in period 1 are given by

$$\frac{u\left(y-\bar{D}_L\right)}{1-\beta}=\begin{cases} \frac{u(0.5y)}{1-\beta}, & \text{if } G^*(\rho_L)\geq \bar{G}_L. \\ u(0.5y)+\frac{\beta u(y-(1-\beta)G^*(\rho_L))}{1-\beta}, & \text{otherwise.} \end{cases}$$

and

$$\frac{u\left(y-(1-\beta)\bar{D}_H\right)}{1-\beta}=\begin{cases} \frac{u(0.5y)}{1-\beta}, & \text{if } G^*(\rho_H)\geq \bar{G}_H. \\ u(0.5y)+\frac{\beta u(y-(1-\beta)G^*(\rho_H))}{1-\beta}, & \text{if } \bar{G}_H > G^*(\rho_H)\geq \bar{G}_L. \\ u(0.5y)+\frac{\beta u(y-(1-\beta)G^*(\rho_H))}{1-\beta}, & \text{otherwise.} \end{cases}$$

Hence the value function is the given by

$$V(D,\rho_H)=\begin{cases} \frac{u(0.5y)}{1-\beta}, & \text{if } G^*(\rho_H)\geq \bar{G}_H \text{ and } D>\bar{D}_H. \\ u(0.5y)+\frac{\beta u(y-(1-\beta)G^*(\rho_H))}{1-\beta}, & \text{if } \bar{G}_H > G^*(\rho_H)\geq \bar{G}_L \text{ and } D>\bar{D}_H. \\ u(0.5y)+\frac{\beta u(y-(1-\beta)G^*(\rho_H))}{1-\beta}, & \text{if } G^*(\rho_H)\leq \bar{G}_L \text{ and } D>\bar{D}_H. \\ \frac{u(y-(1-\beta)D)}{1-\beta}, & \text{if } D\leq \bar{D}_H. \end{cases}$$

and under the low belief regime,

$$V(D,\rho_L)=\begin{cases} \frac{u(0.5y)}{1-\beta}, & \text{if } G^*(\rho_H)>\bar{G}_L \text{ and } D>\bar{D}_L. \\ u(0.5y)+\frac{\beta u(y-(1-\beta)G^*(\rho_L))}{1-\beta}, & \text{if } G^*(\rho_L)\leq \bar{G}_L \text{ and } D>\bar{D}_L. \\ \frac{u(y-(1-\beta)D)}{1-\beta}, & \text{if } D\leq \bar{D}_L. \end{cases}$$

Recall that the thresholds for LT debt are given by

$$\frac{u\left(y-\delta \bar{G}_{L,\delta}\right)}{1-\beta}=\frac{u(0.5y)}{1-\beta}$$

and

$$\frac{u\left(y-\delta_1 \bar{G}_{H,\delta}+q\left[\bar{G}_{H,\delta}-(1-\delta_1)\bar{G}_{H,\delta}\right]\right)}{1-\beta}=\frac{u\left(y-\delta_1[1-q]\bar{G}_{H,\delta}\right)}{1-\beta}=\frac{u\left(0.5y\right)}{1-\beta}$$

Denote the solution to the bargaining problem by $G^*_\delta(\rho_H)$ and $G^*_\delta(\rho_L)$ then the thresholds for default in period 1 are given by

$$\frac{u\left(y-\delta \bar{D}_{L,\delta}\right)}{1-\beta}=\begin{cases}\frac{u(0.5y)}{1-\beta}, & \text{if } G^*_\delta(\rho_L)\geq \bar{G}_{L,\delta}.\\ u(0.5y)+\frac{\beta u(y-\delta(1-q)G^*_\delta(\rho_L))}{1-\beta}, & \text{otherwise.}\end{cases}$$

and

$$\frac{u\left(y-\delta_1[1-q]\bar{D}_{H,\delta}\right)}{1-\beta}=\begin{cases}\frac{u(0.5y)}{1-\beta}, & \text{if } G^*_\delta(\rho_H)\geq \bar{G}_{H,\delta}.\\ u(0.5y)+\frac{\beta u(y-\delta(1-q)G^*_\delta(\rho_H))}{1-\beta}, & \text{if } \bar{G}_{H,\delta}>G^*_\delta(\rho_H)\geq \bar{G}_{L,\delta}.\\ u(0.5y)+\frac{\beta u(y-\delta(1-q)G^*_\delta(\rho_H))}{1-\beta}, & \text{otherwise.}\end{cases}$$

We have that welfare is given by

$$V_\delta(D,\rho_L)=\begin{cases}\frac{u(0.5y)}{1-\beta}, & \text{if } G^*_\delta(\rho_L)\geq \bar{G}_{L,\delta} \text{ and } D\geq \bar{D}_{L,\delta}.\\ u(0.5y)+\frac{\beta u(y-\delta(1-q)G^*_\delta(\rho_L))}{1-\beta}, & \text{if } G^*_\delta(\rho_L)< \bar{G}_{L,\delta} \text{ and } D\geq \bar{D}_{L,\delta}.\\ \frac{u(y-\delta[1-q]D)}{1-\beta} & \text{otherwise}\end{cases}$$

and

$$V_\delta(D,\rho_H)=\begin{cases}\frac{u(0.5y)}{1-\beta}, & \text{if } G^*_\delta(\rho_H)\geq \bar{G}_{H,\delta} \text{ and } D\geq \bar{D}_{L,\delta}.\\ u(0.5y)+\frac{\beta u(y-\delta(1-q)G^*_\delta(\rho_H))}{1-\beta}, & \text{if } \bar{G}_{H,\delta}>G^*_\delta(\rho_H)\geq \bar{G}_{L,\delta} \text{ and } D\geq \bar{D}_{L,\delta}.\\ u(0.5y)+\frac{\beta u(y-\delta(1-q)G^*_\delta(\rho_H))}{1-\beta}, & \text{if } \bar{G}_{H,\delta}>G^*_\delta(\rho_H)\geq \bar{G}_{L,\delta} \text{ and } D\geq \bar{D}_{L,\delta}.\\ \frac{u(y-\delta[1-q]D)}{1-\beta} & \text{otherwise}\end{cases}$$

We know from the previous proposition that $\bar{G}_L<\bar{G}_{L,\delta}$ and $\bar{G}_H<\bar{G}_{H,\delta}$, given that the ST debt case is simply the special case where $\delta=1$. Further, we also know that $G^*(\rho)\leq G^*_\delta(\rho)$.

Having defined and set up the necessary objects to compute the value functions, we now complete the charaterization by computing the equilibrium settlement functions $G^*_{H,\delta}$, $G^*_{L,\delta}$, G^*_H, and $G^*_{L,\delta}$, where the first two denote settlement functions with LT debt under the high and low belief regimes, and the last two are the corresponding functions with ST debt. The key equilibrium condition that determines the equilibrium settlement with LT debt is given by

$$\alpha\{y-\delta(1-q)\tilde{G}_\delta\}+\lambda = (1-\alpha) \Rightarrow \frac{\alpha}{1-\alpha-\lambda} = \frac{1}{y-\delta(1-q)\tilde{G}_\delta} \Rightarrow \lambda > 0, G^*_\delta = \bar{G} < \tilde{G}_\delta, or \lambda = 0, G^*_\delta = \tilde{G}_\delta$$

Similarly, the equilibrium condition that determines the equilibrium settlement function with ST debt is given by

$$\alpha\{y-(1-\beta)\tilde{G}\}+\lambda = (1-\alpha) \Rightarrow \frac{\alpha}{1-\alpha-\lambda} = \frac{1}{y-(1-\beta)\tilde{G}} \Rightarrow \lambda > 0, G^* = \bar{G} < \tilde{G}, or \lambda = 0, G^* = \tilde{G}$$

Now we have that if the constraints do not bind, the unconstrained settlement functions that solve the optimality conditions above are given by

$$\tilde{G}_\delta = \frac{1}{\delta(1-q)}\left\{y - \frac{1-\alpha}{\alpha}\right\}, \quad \tilde{G} = \frac{1}{1-\beta}\left\{y - \frac{1-\alpha}{\alpha}\right\}$$

Now the constraints are given by

$$\bar{G}_H = \kappa y/(1-\beta), \bar{G}_L = \kappa y, \bar{G}_{H,\delta} = \kappa y/\delta(1-q), \bar{G}_{L,\delta} = \kappa y/\delta$$

Then we have that in the absence of rollover risk, under the low belief regime

$$\begin{aligned}
&\tilde{G}_\delta < \bar{G}_{H,\delta} \Leftrightarrow \delta(1-q)\tilde{G} = \delta(1-q)\frac{1}{\delta(1-q)}\left\{y - \frac{1-\alpha}{\alpha}\right\} = \left\{y - \frac{1-\alpha}{\alpha}\right\} < \kappa y, \Leftrightarrow \\
&(1-\beta)\frac{1}{1-\beta}\left\{y - \frac{1-\alpha}{\alpha}\right\} < \kappa y \Leftrightarrow \tilde{G} < \bar{G}_H
\end{aligned}$$

However, in the presence of rollover risk,

$$\begin{aligned}
&\tilde{G}_\delta < \bar{G}_{L,\delta} \Leftrightarrow \delta\tilde{G} = \delta\frac{1}{\delta(1-q)}\left\{y - \frac{1-\alpha}{\alpha}\right\} = \frac{1}{(1-q)}\left\{y - \frac{1-\alpha}{\alpha}\right\} < \kappa y, \\
&\Leftarrow \frac{1}{1-\beta}\left\{y - \frac{1-\alpha}{\alpha}\right\} < \kappa y \Leftrightarrow \tilde{G} < \bar{G}_L
\end{aligned}$$

because

$$1-q > 1-\beta \Rightarrow \frac{1}{1-q} < \frac{1}{1-\beta}$$

But it does not work in the other direction. Hence even after accounting for the differences in the unconstrained solutions, the default constraint binds more with LT debt than with ST debt, especially when the rollover risk is severe (i.e. the low belief regime). In the high belief regime case, reentry decisions are the same across maturities, but not so with the low belief regime. Under the low belief regime, if the sovereign reenters with short-term debt, it also reenters with long-term debt but not the other way around: this is because the difference in the cutoffs under the low belief regime is larger than the cutoffs in the unconstrained solutions, so while the sovereign has to repay a larger settlement with long-term debt, given that it only services a fraction of its debt, they regain market access with greater probability than with short-term debt.

Hence in the case of no rollover risk, and $\{\alpha, \beta, y\}$ is such that $\tilde{G} < \bar{G}_H$ we have

$$V(D, \rho_H) = \begin{cases} \frac{u(0.5y)}{1-\beta}, & \text{if } G^*(\rho_H) \geq \bar{G}_H \text{ and } D > \bar{D}_H. \\ u(0.5y) + \frac{\beta u(y-(1-\beta)G^*(\rho_H))}{1-\beta} = u(0.5y) + \frac{\beta u(\frac{1-\alpha}{\alpha})}{1-\beta}, & \text{if } \bar{G}_H > G^*(\rho_H) \text{ and } D > \bar{D}_H. \\ \frac{u(y-(1-\beta)D)}{1-\beta}, & \text{if } D \leq \bar{D}_H. \end{cases}$$

and similarly

$$V_\delta(D, \rho_H) = \begin{cases} \frac{u(0.5y)}{1-\beta}, & \text{if } G^*_\delta(\rho_H) \geq \bar{G}_{H,\delta} \text{ and } D \geq \bar{D}_{H,\delta}. \\ u(0.5y) + \frac{\beta u(y-\delta(1-q)G^*_\delta(\rho_H))}{1-\beta} = u(0.5y) + \frac{\beta u(\frac{1-\alpha}{\alpha})}{1-\beta}, & \text{if } \bar{G}_{H,\delta} > G^*_\delta(\rho_H) \text{ and } D \geq \bar{D}_{H,\delta}. \\ \frac{u(y-\delta[1-q]D)}{1-\beta} & \text{otherwise} \end{cases}$$

Hence we have that

$$V_\delta(D, \rho_H) - V(D, \rho_H) = \begin{cases} 0, & \bar{D}_H \leq \bar{D}_{H,\delta} \leq D \\ \frac{u(y-\delta[1-q]D)}{1-\beta} - u(0.5y) - \frac{\beta u(\frac{1-\alpha}{\alpha})}{1-\beta} & \bar{D}_H \leq D \leq \bar{D}_{H,\delta} \\ \frac{u(y-\delta[1-q]D)}{1-\beta} - \frac{u(y-(1-\beta)D)}{1-\beta} & D \leq \bar{D}_H \leq \bar{D}_{H,\delta} \end{cases}$$

Alternatively, if $\tilde{G}_\delta > \bar{G}_H$ then proceeding as above,

$$V_\delta(D, \rho_H) - V(D, \rho_H) = \begin{cases} 0, & \bar{D}_H \leq \bar{D}_{H,\delta} \leq D \\ \frac{u(y-\delta[1-q]D)}{1-\beta} - \frac{u(0.5y)}{1-\beta} & \bar{D}_H \leq D \leq \bar{D}_{H,\delta} \\ \frac{u(y-\delta[1-q]D)}{1-\beta} - \frac{u(y-(1-\beta)D)}{1-\beta} & D \leq \bar{D}_H \leq \bar{D}_{H,\delta} \end{cases}$$

By contrast, under the low belief regime, one has to consider three cases. First, when both $\tilde{G}_\delta < \bar{G}_{L,\delta}$ and $\tilde{G} < \bar{G}_L$

$$V_\delta(D, \rho_L) - V(D, \rho_L) = \begin{cases} 0, & \bar{D}_L \leq \bar{D}_{L,\delta} \leq D \\ \frac{u(y-\delta[1-q]D)}{1-\beta} - u(0.5y) - \frac{\beta u(\frac{1-\alpha}{\alpha})}{1-\beta} & \bar{D}_L \leq D \leq \bar{D}_{L,\delta} \\ \frac{u(y-\delta[1-q]D)}{1-\beta} - \frac{u(y-(1-\beta)D)}{1-\beta} & D \leq \bar{D}_L \leq \bar{D}_{L,\delta} \end{cases}$$

whereas if $\tilde{G}_\delta < \bar{G}_{L,\delta}$ and $\tilde{G} > \bar{G}_L$

$$V_\delta(D, \rho_L) - V(D, \rho_L) = \begin{cases} u(0.5y) + \frac{\beta u(\frac{1-\alpha}{\alpha})}{1-\beta} - \frac{u(0.5y)}{1-\beta}, & \bar{D}_L \leq \bar{D}_{L,\delta} \leq D \\ \frac{u(y-\delta[1-q]D)}{1-\beta} - \frac{u(0.5y)}{1-\beta} & \bar{D}_L \leq D \leq \bar{D}_{L,\delta} \\ \frac{u(y-\delta[1-q]D)}{1-\beta} - \frac{u(y-(1-\beta)D)}{1-\beta} & D \leq \bar{D}_L \leq \bar{D}_{L,\delta} \end{cases}$$

and finally if $\tilde{G}_\delta > \bar{G}_{L,\delta}$ and $\tilde{G} > \bar{G}_L$

$$V_\delta(D,\rho_L) - V(D,\rho_L) = \begin{cases} 0, & \bar{D}_L \leq \bar{D}_{L,\delta} \leq D \\ \frac{u(y-\delta[1-q]D)}{1-\beta} - u(0.5y) - \frac{\beta u(\frac{1-\alpha}{\alpha})}{1-\beta} & \bar{D}_L \leq D \leq \bar{D}_{L,\delta} \\ \frac{u(y-\delta[1-q]D)}{1-\beta} - \frac{u(y-(1-\beta)D)}{1-\beta} & D \leq \bar{D}_L \leq \bar{D}_{L,\delta} \end{cases}$$

The difference in going from the low to the high belief regime with LT debt and $\tilde{G}_\delta < \bar{G}_{L,\delta} < \bar{G}_{H,\delta}$ is given by

$$V_\delta(D,\rho_H) - V_\delta(D,\rho_L) = \begin{cases} \frac{u(y-\delta[1-q]D)}{1-\beta} - u(0.5y) - \frac{\beta u(\frac{1-\alpha}{\alpha})}{1-\beta} & \bar{D}_{L,\delta} \leq D \leq \bar{D}_{H,\delta} \\ 0 & \text{otherwise} \end{cases}$$

Finally, if we consider the most interesting case $\tilde{G}_\delta < \bar{G}_{L,\delta} < \bar{G}_{H,\delta}$ and $\bar{G}_H > \tilde{G} > \bar{G}_L$ and compute instead the difference of going from the low belief regime to the high belief regime and using LT debt instead of ST debt, one obtains,

$$V_\delta(D,\rho_H) - V(D,\rho_L) = \begin{cases} u(0.5y) + \frac{\beta u(\frac{1-\alpha}{\alpha})}{1-\beta} - \frac{u(0.5y)}{1-\beta}, & \bar{D}_{H,\delta} \leq D \\ \frac{u(y-\delta[1-q]D)}{1-\beta} - u(0.5y) - \frac{\beta u(\frac{1-\alpha}{\alpha})}{1-\beta} + u(0.5y) + \frac{\beta u(\frac{1-\alpha}{\alpha})}{1-\beta} - \frac{u(0.5y)}{1-\beta} & \bar{D}_{L,\delta} \leq D \leq \bar{D}_{H,\delta} \\ \frac{u(y-\delta[1-q]D)}{1-\beta} - \frac{u(0.5y)}{1-\beta} & \bar{D}_L \leq D \leq \bar{D}_{L,\delta} \\ \frac{u(y-\delta[1-q]D)}{1-\beta} - \frac{u(y-(1-\beta)D)}{1-\beta} & D \leq \bar{D}_L \end{cases}$$

as desired. □

Hence there are four big differences between the ST and LT case that make longer maturity welfare superior: (1) there is less default due to $\bar{D}_{L,\delta} > \bar{D}_L$ and $\bar{D}_{H,\delta} > \bar{D}_H$, (2) there is greater steady state consumption due to lower debt service, even when the sovereign repays in full in both cases, (3) there is faster reentry due to $\bar{G}_{L,\delta} > \tilde{G}_\delta$ and $\tilde{G} > \bar{G}_L$, (4) there are also gains from lower debt service after restructuring and reentry, as the settlement is less than the output cost of default. To understand what this means, think back to the case with exogenous settlement. If the exogenous settlement rule is high enough that the country will choose to stay in default instead of reentering immediately regardless of the belief regime, only the first two sources of welfare gains above are operational. If instead the exogenous settlement rule is such that the country chooses to repay and reenter immediately rather than staying in default with LT debt, then all four sources of welfare gains are operational and LT debt does not only reduce the probability of initial default, but also reduces the delays associated with restructuring and also gains from the higher consumption resulting from less debt service and being in good standing. The proof above shows that this property continues to hold even when settlement is endogenous, where in the low belief regime with rollover risk, if the country reenters with ST debt, the country also reenters with LT debt but not the other way around, implying fewer delays and inefficient losses from self-fulfilling debt restructuring due to LT debt.

5 Calibration

To perform quantitative analysis and test our model predictions against data, we need to specify functional forms to preferences and technoogies, assign values to structural parameters, and make assumptions about the structure of the underlying stochastic processes governing the shocks. Consistent with the quantitative literature on sovereign debt, we adopt CRRA preferences for the borrower, with a coeffcieint of risk aversion of 2. The risk-free rate at which lenders can borrow is set at one percent, and the length of a period is a quarter.

I adopt a two-pronged approach in calibrating the rest of the parameters to match Argentinian data. On the one hand, given the focus of this paper on restructuring, I target five key moments of the restructuring process using five parameters that strongly affect the model's predictions along these dimnesions. On the other hand, solving the model and computing the equilibrium also requires parameterizing the rest of the model and this is done in parallel, using data largely orthogonal to the restructuring moments. More specifically, the five moments I target are the following: (1) the average haircut, (2) the standard deviation of the haircut, (3) the average debt to GDP ratio, (4) the mean default frequency, and the (5) mean spread. To hit these targets, I vary the values for the following parameters: (1) the transition probability for the high belief state π_h, (2) the transition probability for the low belief state π_l, (3) discount factor β, (4) output cost parameters a_0, and (5) a_1. The output cost parameters are part of an output cost process

$$L(y) = \max(0, a_1 y_t + a_2 y_t^2)$$

The parameters governing the output cost of default is useful to generating the right default frequency and in this environment with risk-neutral investors, the mean spread. The two transition probabilities for beliefs are part of a two-state first-order Markov process assumed for beliefs, where π_h is the probability that beliefs transition from a high state today to a high state tomorrow while π_l is the probability that beliefs today transition from a low state to a low state in the next period. Consistent with the key message of this paper, the beliefs are used to discipline the moments related to restructuring: both the size and volatility of the haircuts. The discount factor accounts for the average level of borrowing as the borrower's impatience motivates it to continue borrowing in spite of the increasing cost of doing so. As seen in the table below, the model does a good job of matching these key moments: the model predicts an average haircut of 21% to an average of 22% in the data; the standard deviation of haircuts predicted by the model (34) only slightly underestimates that observed in the data (40). This is a substantial improvement to the model with no difference in beliefs, where the standard deviation in haircuts is significantly underestimated (5). The model also comes close to the data in the other three moments related to default and restructuring: it predicts a slightly higher debt to GDP ratio, which translates to a higher default frequency, and hence risk premia.

Table 1: Baseline Parameterization

Parameter	Value	Target or Source
σ	2	Yue (2010)
λ	0.05	Broner et al. (2013)
r	0.01	US Treasury-bill interest rate
z	0.03	annual coupon rate
ξ	0.07	years to settlement
θ	0.9	years in exclusion
a_1	0.15	average spread
a_2	-0.225	stdev spread
β	0.96	debt-to-GDP ratio
π	0.7	stdev haircut
α	-0.025	mean haircut

To complete the discussion of the parameterization, I briefly describe the values assigned to the rest of the parameters. These numbers are consistent with numbers typically used in the literature. The probability of reentry θ is calibrated to allow the model to replicate the length of exclusion observed in the data. Note that in this model environment, this parameter does not have complete control over the duration of exclusion; countries can endogenously choose to remain in exclusion by restructuring even when given the opportunity to reenter hence placing the inverse of this parameter at the lower bound of the exclusion duration. The output process is estimated using data on Argentinian tradable output, and estimates are similar to those used in Uribe. I consider a variant of the calibration above, where the bargaining power α is estimated in lieu of a_1 (where $a_1 = 1$, for a flat output cost) and this is justified by the same means, as the risk premium in this case does not only reflect the default frequency but the size of the haircut as well and α is good for hitting the average haircut. Alternatively, if a_1 is allowed to vary, we can do sensitivity analysis on α as this has significant implications for the quantitative results. Note finally that other desirable cyclical properties of the model without beliefs that are consistent with the data (e.g. the positive correlation between spreads and the trade balance, and the countercyclicality of the spread) are both preserved in the model with beliefs. Thel list of parameter values are found in Table 1.

6 Quantitative Analysis

I now discuss the quantitative properties and implications of the calibrated model. To test the model's quantitative implications, we compare model statistics with data statistics for Argentina. The bond spreads data are from JP Morgan's Emerging Bond Indices (EMBI) dataset for Argentina (the spreads are computed relative to US treasury bonds of similar duration). MECON is the source of consumption and trade data, which are quarterly and seasonally adjusted. The measure of external debt is obtained from the World Bank's Global Development Finance dataset. Data for haircuts and debt restructuring are obtained from Cruces and Trebesch (2013), and calculated

Table 2: Model Statistics: Comparison with Other (Recalibrated) Models

Statistic	Data	Baseline	Output Risk Only	No Restructuring
$E(r-r^*)$	0.07	0.07	0.09	0.07
$\sigma(r-r^*)$	0.05	0.07	0.05	0.06
$\text{corr}(r-r^*, y)$	-0.64	-0.20	-0.30	-0.11
$\text{corr}(r-r^*, tb/y)$	0.72	0.80	0.85	0.50
$E(d/y)$	1.0	1.2	0.8	0.1
$E(h)$	0.4	0.4	0.6	0.0
$\sigma(h)$	0.24	0.22	0.07	0.0
$E(L)$	15	15	10	8

using the methodology using Sturzenegger and Zettelmeyer (2006).

The model matches the key data statistics. In particular, it generates sufficient heterogeneity in the fraction of debt recovered, a result of both having restructurings from both fundamental and non-fundamental sources. By contrast, the model with only output (fundeamental) risk can only generate a fraction of the required volatility (standard deviation of 7). Both these models can account for the average size of debt recovered; in the current parameterization, our baseline with both types of risks predicts a slightly higher level, but only because this moment is simultaneously targeted with the volatility of haircuts, which the model hits perfectly. The expected length in exclusion is also best matched by the model with both types of risk, signifying that the endogenous delays emphasized earlier are quantitatively important. All things considered, this is evidence that the self-fulfilling dynamics highlighted our baseline model coupled with output risk successfully generates the variation in restructurings previously unaddressed.

To generate the debt-to-GDP ratios and spread statistics observed empirically, modern quantitative models of default require both LT debt and a specific parameterization for the output costs of default. The addition of belief regimes do not affect the standard model's ability to successfully match these moments though the variation in the spread is slightly higher than that observed in the data. This is a result of greater rollover risk generating greater default and with it greater variation in the spread. It also speaks to the difficulty of matching both spread and restructuring moments at the same time, when the model's mechanism generates variation in the restructuring endogenously through variation in the spread. In terms of business cycle statistics, the model's predictions are broadly in line with the data: it predicts consumption that is more volatile than output, and countercyclical trade balance that results from the bond price that reflects the default risk that arises from low output and pessimistic lenders. Our model shares these predictions with other standard models.

6.1 Role of Beliefs

In this section, we delve deeper into the role of the belief regime for generating empirically relevant spreads and haircuts. In our benchmark specification, we target both the mean and volatility of the haircuts and show that when beliefs are such that on average, half of debt is recovered but that the amount recovered varies just as much as in the data, the implied spreads are both higher in levels on average, and vary more than in the data. To understand this, note that the channel through which heterogeneous haircuts are generated in the model is through spreads: when spreads rise as a result of low output or pessimistic beliefs, the value of repayment falls because the price that debt carries falls dampening consumption in repayment and hence inducing default and restructuring. The key thing is that spreads not only change the probability of restructuring, but also the severity of the restructuring event: when prices are low and the value of repayment is low, the amount of debt that can be recovered also falls; conversely, when prices are relatively high and the value of repayment high, more debt can be recovered. This implies that the volatility in spreads imply a volatility in haircuts and that these two stochastic processes are not independent, but rather, are closely intertwined and jointly determined by the model.

The next table shows that for an alternative specification of beliefs, the model can match the level and volatility of spreads. In this alternative specification, the model predicts haircuts that are substantially less volatile than empirically observed, though the mean level of debt recovered produced remains close to its the data counterpart. This is driven in part by the bargaining power, which ensures that the surplus from restructuring is divided proportionally, implying that a certain amount of debt can be recovered. We explore the role of the bargaining power parameter in greater depth in the next section.

Holding fixed the rest of the model parameters, we see that varying the transition probability of going from the low-belief regime to the high-belief regime $1 - \pi_l$ has a stronger impact on the volatility of debt recovery than varying the transition probability of going from the high-belief regime to the low-belief regime $1 - \pi_h$. To understand this, first note that in the limiting case where the high belief regime is an absorbing state, once the high belief regime is realized, the model devolves into the model with only output shocks, implying similar volatilities for both spreads and haircuts. This is indeed what we observe: in the cases where the transition probability of going from the high belief regime to the low belief regime approaches one, the statistics generated are very similar to the model with only output shocks.

If a permanent high belief regime generates model statistics that mimic those produced by a model with only output risk, why does parameterization with a permanent low belief regime generate model statistics that differ drastically from the model with only output risk? The reason is that the crisis zone only really comes into play under the low belief regime, so that there is default and restructuring for a larger fraction of states implying a greater degree of heterogeneity in bargaining conditions, and hence outcomes. This is because in the crisis zone, prices no longer reflect only fundemantal (i.e output) considerations but also contain a self-fulfilling component that pushes it down and creates a gap between the pricing function that also applies in the environment

Table 3: Model Statistics: Different Beliefs

(π_H, π_L)	Baseline	(0.4,0.4)	(0.1,0.1)	(0.7,.0.1)	(0.1,0.7)
$E(d/y)$	1.2	1.3	1.6	1.7	0.8
$E(r-r^*)$	0.07	0.07	0.01	0.01	0.21
$\sigma(r-r^*)$	0.07	0.06	0.003	0.003	0.31
$E(h)$	0.4	0.3	0.09	0.08	0.64
$\sigma(h)$	0.22	0.15	0.04	0.02	0.23

with only output risk and the more general pricing function that takes both output and rollover risk into account. Hence under the low belief regime both prices and haircuts are more volatile than under the high belief regime, and to generate the required volatility in haircuts, what matters more is to have a sufficiently persistent low belief regime, as opposed to a persistent high belief regime.

The high belief regime would only be sufficient if output risk alone were sufficient, however, this is not the case as the macro aggregates associated with restructuring vary more than that implied by output risk alone. Furthermore, we also observe from the table that while the low belief regime has to be persistent to generate sufficient variability (hence the importance of self-fulfilling debt restructuring due to rollover risk), the high belief regime also has have some persistence (as we have in our baseline parameterization) because if the high belief regime is temporary, the sovereign spends too much time in the low belief regime and this generates too much variation (see the last column). Hence the belief process the data identifies is one wherein the current belief regime the country finds itself in is informative for the future, regardless of whether it finds itself in the high or low belief regimes, because the states are persistent but not perfectly persistent, and only such a stochastic process can generate the empirically observed regularities.

6.2 Role of Bargaining

In this section, we investigate the role of the bargaining power parameter in accounting for debt renegotiation outcomes in the presence of both fundamental and non-fundamental risks. With only output or fundamental risk, bargaining power is the key parameter controlling the bargaining protocol and hence, the debt recovery rate. In such an envronment, varying bargaining power has a non-monotonic effect on spreads and default frequencies because of two opposing forces. The direct effect of raising the borrower's bargaining power lowers the amount of debt recovered keeping everything constant, as the borrower's value is decreasing in the amount of recovered debt; this tends to lower debt prices and raise the probability of default. In general equilibrium, this decrease in the amount of debt recovered implies less debt is repaid, and the lower debt price in turn reduces the incentive to issue debt. This general equilibrium effect of issuing less debt goes in the opposite direction as less debt implies less default and higher prices. Which effect dominates then determines whether more or less debt is recovered as more and more bargaining power is accorded

Table 4: Model Statistics: Different Effective Bargaining Power

α	-0.05	Baseline	-0.01	0.01	0.05
$E(d/y)$	1.9	1.2	0.7	0.6	0.6
$E(r-r^*)$	0.01	0.07	0.29	0.48	0.49
$\sigma(r-r^*)$	0.01	0.07	0.56	1.04	1.05
$E(h)$	0.04	0.4	0.8	0.8	0.9
$\sigma(h)$	0.06	0.22	0.23	0.23	0.23

to the borrower. In an environment with only output risk, Yue (2010) finds that this results in non-monotonic changes in the spreads and default frequency as the bargaining power parameter goes from zero to one.

In this envrionment with both belief and output shocks and flexible bargaining, we find that for a variety of alternative belief parameterizations that the first (direct) effect mentioned above dominates and default probabilities fall and spreads rise as the outside option parameter α (which acts like an effective bargaining power) increases and the haircut goes to one. The effect on the amount of debt recovered also falls with increasing effective bargaining power, just as Yue (2010) finds. What is more interesting is the fact that in the four different parameterizations considered, the volatility of spreads increases monotonically with the outside option even as the volatiility of haircuts increase initially before tapering off. The reason why volaility in haircuts cannot possibly increase monotonically is because haircuts is a ratio that can only take on a value between 0 and 1, as opposed to spreads which are unbounded. This is informative because given a particular (π_h, π_l) pair, while changing the outside option parameter α can change both the average and variance of haircuts, this alone will not be sufficient to guarantee that one can generate sufficient volatility in haircuts or debt recovery rates; a combination of both the right type of persistence in the belief regime (as discussed above) and outside option or bargaining power is required to deliver the required volatilities not just in the restructuring outcomes, but in the spreads as well.

6.3 Debt Maturity, Self-Fulfilling Debt Restructuring, and Welfare

The quantitative literature on sovereign debt has emphasized the importance of long-term debt in allowing the Eaton-Gersovitz class of models to match the data moments for the key pre-default aggregates (debt-to-GDP ratio, mean and standard deviation of spreads, etc). I find that consistent with Chatterjee and Eyigungor (2009) and Hatchondo and Martinez (2009), the performance of the model in terms of matching the standard moments on spreads and levl of sustainable debt suffers when the sovereign can only use short-term debt to borrow from financial markets. The spreads in particular falls dramatically both in terms of mean and variance. Debt-to-GDP does not fall drastically as there is both output and rollover risk in this environment, and the presence of both types of risks create incentives for countries to hold larger amounts of debt and default more often in equilibrium. The restructuring aggregates are affected in a similar manner and the model with

Table 5: Debt Maturity, Self-Fulfilling Debt Restructuring, and Welfare (Consumption Equivalent)

$\alpha = -0.025$			
(π_h, π_l)	(0.7,0.7)	(0.95,0.05)	(0.05,0.95)
ST Debt	0.985	1.033	0.945
LT Debt	0.997	1.039	0.972
Δc	0.012	0.006	0.027
$\alpha = 0.025$			
(π_h, π_l)	(0.7,0.7)	(0.95,0.05)	(0.05,0.95)
ST Debt	0.969	1.008	0.945
LT Debt	0.980	1.005	0.969
Δc	0.011	-0.003	0.024

long-term debt matches the restructuring data better than its short-term counterpart.

Beyond the positive implications of introducing long-term debt into our model of self-fulfilling debt restructuring, the costly delays and output lost during restructuring imply that self-fulfilling debt restructuring has important normative ramifications as well. We show in the section dedicated to understanding the mechanism that aside from helping the sovereign circumvent default by lowering debt service requirements, by mitigating rollover risk, long-term debt can also help reduce the costly delays associated with self-fulfilling debt restructuring. The next table computes the consumption equivalent difference when one moves from an environment with ST debt to one with LT debt under three different belief regimes: the baseline parameterization where both high and low belief regimes are fairly persistent ($\pi_h = \pi_l = 0.7$), the regime with minimal rollover risk ($\pi_h - 0.95, \pi_l = 0.05$), and the regime with maximal rollover risk ($\pi_h - 0.05, \pi_l = 0.95$). We also vary the borrower's outside option α to see how rollover risk interacts with debt restructuring (to really understand the extent of self-fulfilling debt restructuring). In accordance with the qualitative predictions of the simplified model discussed before, we find that LT debt improves welfare most in the environment where rollover risk is most severe ($\Delta c = 0.027$). Note that this result is in the context of the full blown quantitative model with both output and rollover risk, in contrast to the simplified model that can be characterized analytically and only had rollover risk. Further, note that quantatively, the effect of maturity choice in the environment with severe rollover risk is more than four times the welfare impact in the case with minimal rollover risk ($\Delta c = 0.006$). We see that this result is robust for an alternative bargaining game where the outside option for the borrower is raised to $\alpha = 0.025$. With a greater bargaining power, the sovereign defaults more and we find that output risk becomes more important in the case with minimal rollover risk, where we observe that switching to LT debt lowers welfare due to the debt dilution problem. In all but this extreme case, however, long-term debt is Pareto-superior to ST debt and rollover risk is more important than output risk for welfare, especially in the environment where it has a significant impact on the restructuring process.

7 Conclusion

Sovereign debt crises and default have long had a significant impact on borrowing countries and international capital markets. Much progress has been made toward understanding why countries default and how tor regain market access. Less well-understood is how countries restructure their debt following default, and what causes these restructuring events to have such different outcomes. There is huge variation in the fraction of debt recovered, as well as the length in exclusion during these debt restructuring episodes, and worse outcomes (longer exclusion duration and lower debt recovery) tend to be associated with higher spreads during the restructuring. This paper provides a framework for understanding these heterogeneous debt restructuring experiences by embedding sovereign debt restructuring in a model with both output and rollover risk. In the model, when lenders are optimistic and rollover risk is high, spreads are lower, and restructuring outcomes better (more debt recovered, shorter exclusion duration) as the lower spreads raise the value of repayment relative to autarky, raising the borrower's surplus and allowing more debt to be recovered from the bargaining protocol. Standard models rely only on output risk so that this self-fulfilling channel is not operational; these models can only account for a fraction of the volatility in haircuts observed in the data. By contrast, this model with output and belief shocks can successfully generate the volatility in haircuts and exclusion duration observed in the data. Rollover risk also plays a larger role in accounting for debt and spread dynamics in this environment because its impact on the restructuring process amplifies the effect it has on dynamics leading up to a self-fulfilling debt crisis. Finally, the welfare implications of allowing for self-fulfilling debt restructuring are quantitatively significant. Our results show that not only do the welfare gains from mitigating rollover risk outweight the welfare losses from debt dilution in this model with long-term debt, but that the consumption-equivalent measure of welfare can be more than ten percent higher if lenders are persistently optimistic instead of being persistently pessimistic. These findings all point to the importance of self-fulfilling lending expectations in accounting for sovereign debt, default, and restructuring dynamics.

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8 Appendix A: Three-Period Example

To help build intuition, I now present a simplified, finite period version of the model that can be characterized analytically with the right set of conditions. There are three periods, $t = 1, 2, 3$. Assume that output can only be low or high, and that the realization of the shocks is (L, H, H), in that order. The belief regimes, for simplicity, are assumed to be permanent; this is relaxed in the quantitative analysis and the case with a non-degenerate transition matrix is discussed at length in a separate section later. Default cost κ is assumed to be a utility cost to keep things tractable, and log utility is assumed. The restructured debt is settled in the subsequent period if the country gets the chance to reenter and chooses to do so (i.e. repay the original debt net of haircut rather than go through another restructuring.) The probability of reentering after default is 1; this, however, does not imply that the country necessarily reenters the period after it defaults, as it can always restructure yet again. There is no further restructuring in the last period (remember the finite-period simplifying assumption) and debt then must be repaid. The renegotiation process follows the Nash bargaining protocol specified earlier with bargaining power θ. For simplicity, assume that the discount factor $\beta = 1$ and the risk-free interest rate is zero. Assume that the initial conditons are such that the country finds itself in the crisis zone even after its debt has already been restructured once (sufficient conditions for this can be provided). I will now show that given the sequence of output shocks, the belief regime matters along at least two key dimensions: the amount of debt restructured, and the length of exclusion. In particular, the following proposition shows that the high belief regime yields a higher amount of debt recovered, and the country spends fewer periods in exclusion. The content of this proposition is that, by introducing different belief regimes, we can raise the volatility not only of spreads but also of both haircuts and the length in exclusion, and this is important because while we observe significant variation in both haircuts and exclusion duration in the data, the standard model without beliefs is unable to generate this. Furthermore, the two results embedded in this proposition are also in line with the motivating evidence presented in Cruces-Trebesch (2013): longer exclusion duration and larger haircuts are associated with higher spreads, which is exactly what the model would predict given a low belief regime.

The proof of the proposition requires the use of the Arithmetic Mean-Geometric Mean Inequality (AM-GM), which states that

Arithmetic Mean-Geometric-Mean Inequality.
For any list of n non-negative real numbers $x_1, x_2, \ldots, x_n$, we have

$$\frac{x_1 + x_2 + \ldots + x_n}{n} \geq \sqrt[n]{x_1 x_2 \ldots x_n} \tag{1}$$

with equality if and only if $x_1 = x_2 = \ldots = x_n$.

There are various different ways of proving this inequality (e.g. using mathematical induction), below I present a short proof that requires the use of Jensen's inequality (a more familiar inequality that has found use in many economic applications).

Proof. If $x_1 = x_2 = \ldots = x_n$ holds then the required equality is easily verified. Now consider the case where it does not hold. Given that log is strictly concave, we have by Jensen's inequality that

$$\log\left(\frac{x_1 + x_2 + \ldots + x_n}{n}\right) > \frac{\log(x_1) + \log(x_2) + \ldots + \log(x_n)}{n} = \log\left(\sqrt[n]{x_1 x_2 \ldots x_n}\right) \tag{2}$$

then given that log is strictly increasing, the required inequality follows:

$$\frac{x_1 + x_2 + \ldots + x_n}{n} > \sqrt[n]{x_1 x_2 \ldots x_n} \tag{3}$$

With this in hand, let me now proceed to the main proposition:

Proposition 1. Debt recovery rates are higher and exclusion durations shorter in the high (optimistic) belief regime, compared to the low (pessimistic) belief regime.

Proof. To compute debt recovery rates, one needs to compute the surpluses necesary for solving the bargaining problem. The borrower's surplus is the difference between the value of repayment (given the restructuring) and the value in autarky, where the value in autarky is given by

$$V_a = \log(y_L) - \log(\kappa) + \log(y_H) - \log(\kappa) + \log(y_H) - \log(\kappa) \tag{4}$$

The value in repayment varies with the belief regime. Under the low belief regime, the country not only restructures in the first period, but restructures in the second period as well (recall the assumption of being in the crisis zone after one restructuring). This implies that the value of repayment under the low belief regime is given by

$$V_r(\mathcal{L}) = \log(y_L) - \log(\kappa) + \log(y_H) - \log(\kappa) + \log(y_H - \alpha_L b) \tag{5}$$

By contrast, under the high belief regime, the country only restructures in the first period and regains financial market access in the second period. As such, it is able to issue debt at price 1 in period 2, and it will find it optimal to issue half the amount of recovered debt in order to smooth consumption between periods 2 and 3. This implies that the value of repayment for the high belief regime is then given by

$$V_r(\mathcal{H}) = \log(y_L) - \log(\kappa) + \log(y_H - \alpha_H b + \alpha_H b/2) + \log(y_H - \alpha_H b/2) \tag{6}$$

Now form the bargaining problem for the low belief regime. Under the low belief regime, we have the surpluses given by

$$\Delta_b(\mathcal{L}) = V_r(\mathcal{L}) - V_a = \log(y_H - \alpha_l b) - \log(y_H) + \log(\kappa), \quad \Delta_s = \alpha_l b \tag{7}$$

This then implies that

$$\max \Delta_b(\mathcal{L})^\theta \Delta_s^{1-\theta} \tag{8}$$

$$\text{F.}O.C. \quad \frac{\theta}{1-\theta}\frac{\alpha b}{y_l - \alpha b} = \Delta_b = \log(y_l - \alpha b) - \log(y_l) + \log(k) \tag{9}$$

$$\Rightarrow \frac{\theta}{1-\theta} = \frac{y_l - \alpha b}{\alpha b} \log\left[\frac{y_l(y_l - \alpha_l b)\kappa}{y_l^2}\right] \equiv RHS_L \tag{10}$$

Under the high belief regime, the

$$\Delta_b(\mathcal{H}) = V_r(\mathcal{H}) - V_a = 2\log(y_H - \alpha_l b/2) - 2\log(y_H) + 2\log(\kappa), \quad \Delta_s = \alpha_h b \tag{11}$$

Which yields the following optimality condition

$$\max \Delta_b(\mathcal{H})^\theta \Delta_s^{1-\theta} \tag{12}$$

$$\text{F.}O.C. \quad \frac{\theta}{1-\theta}\frac{\alpha b}{y_h - \alpha b/2} = 2\log(y_h - \alpha b/2) - 2\log(y_h) + 2\log(k) \tag{13}$$

$$\Rightarrow \frac{\theta}{1-\theta} = \frac{y_h - \alpha b/2}{\alpha b} \log\left[\frac{(y_h - \alpha_h b/2)\kappa}{y_h}\right]^2 \equiv RHS_H \tag{14}$$

To prove the $\alpha_h > \alpha_l$ first note the following sequence of inequalities

$$\log\left[\frac{(y_h - \alpha_h b/2)\kappa}{y_h}\right]^2 > \log\left[\frac{y_h(y_h - \alpha_h b)\kappa^2}{y_h^2}\right] > \log\left[\frac{y_l(y_l - \alpha_l b)\kappa}{y_l^2}\right] \tag{15}$$

the first inequality above uses the Arithmetic Mean-Geometric-Mean Inequality, while the second inequality follows from having a positive default cost, i.e. $\kappa > 1$. Combining this sequence of inequalities with $y_h - \frac{\alpha b}{2} > y_h - \alpha b$ yields $RHS_H(\alpha) > RHS_L(\alpha)$ for any given α. Now recall from the optimality conditions above that

$$RHS_H(\alpha_h) = RHS_L(\alpha_l) = \frac{\theta}{1-\theta} \tag{16}$$

Finally using the fact that $RHS_H(\alpha)$ and $RHS_L(\alpha)$ are decreasing in α, we have $\alpha_h > \alpha_l$ as desired. To complete the proof, note that the clause regarding exclusion duration is satisfied as well given that the economy finds itself restructuring twice (and hence being in exclusion for 2 periods) in the low belief regime, while it only spends one period in exclusion in the high belief regime. Q.E.D.