Intermediate Microeconomics
Econ 3101, Section 002
Homework 5

Timothy Lim Uy

Question 1. ***Exchange.***
Consider an exchange economy with two goods and two consumers (indexed by i), each with endowments $e_i = (2, 2)$ for $i = 1, 2$.
(a) [10] Find all Pareto-optimal allocations when $u_1(x_1, x_2) = x_1 + 2x_2$ and $u_2(y_1, y_2) = 2y_1 + y_2$.
(b) [15] Find all competitive equilibria when $u_1(x_1, x_2) = x_1 + 2x_2$ and $u_2(y_1, y_2) = 2y_1 + y_2$.

Question 2. ***Welfare.***
Suppose a cataclysmic event occurs and suddenly there are only 101 people left on the planet. You have been declared king (queen) and must now decide what to give to the rest of your (100) subjects. After taking your share, there are only 500 resource units left. Of these 100 people, 70 are old (X) while 30 are young (Y). For simplicity, assume all the old (young) people are treated the same, but that the old and young can be treated differently.
(a) [5] If you have a utilitarian welfare function $U(X, Y) = 70X + 30Y$, how much would the old people get as a group? What about the young?
(b) [5] Suppose you have a Rawlsian welfare function $U(X, Y) = min\{70X, 30Y\}$ instead. What would the old receive as a group? What about the young?
(c) [5] What if $U(X, Y) = (70X)^{0.2}(30Y)^{0.8}$? What do the old (young) get as a result?
(d) [5] Take the concave utility function $U(X, Y) = (70X)^2 + (30Y)^2$. What amount gets allocated to the old? The young? (Hint: There may be more than one optimal outcome.)
(e) [5] Finally consider the Nietzschean welfare function $U(X, Y) = max\{70X, 30Y\}$. How much does each group receive in this case? (Hint: There may be more than one optimal outcome.)

Question 3. ***Asymmetric Information.***
(Varian) In Pot Hole, Georgia, 1000 people want to sell their used cars. These cars vary in quality. Original owners know exactly what their cars are worth. All used cars look the same to potential buyers until they have bought them; then they find out the truth. For any number X between 0 and 2000, the number of cares of quality lower than X is $\frac{X}{2}$. If a car is of quality

X, its original owner will be willing to sell it for any price greater than X. If a buyer knows that a car is of quality X, she will be willing to pay $X + 500$ for it. When buyers are not sure of the quality of a car, they are willing to pay its expected value, given their knowledge of the distribution of qualities on the market.

(a) [10] Suppose that everyone knows that all the used cars in Pot Hole are for sale. What would used cars sell for? Would every used car owner be willing to sell at this price? Which used cars would appear on the market?

(b) [10] Let X^* be some number between 0 and 2000 and suppose that all cars of quality lower than X^* are sold, but original owners keep all cars of qualit higher than X^*. What would buyers be willing to pay for a used car? At this price, which used cars would be for sale?

(c) [5] Write an equation for the equilibrium value of X^*, at which the price that the buyers are willing to pay is exactly enough to induce all cars of quality less than X^* to enter the market. Solve this equation for X^*.

Question 4. ***Auctions.***

Imagine a world where there are three bidders, A, B, and C, and their individual valuations of the object under consideration is H with probability p and L with probability $1 - p$, where $H > L$ and $0 < p < 1$. Ties are broken randomly. All proceeds go to the auctioneer.

(a) [5] If the mechanism is a second-price sealed-bid auction and $p = \frac{2}{3}$, what should A bid when he values the object at H? At L?

(b) [5] Given the information in part (a), if the bidders don't collude, what is the probability that the auctioneer makes H? What will this probability be if $p = \frac{1}{2}$ instead?

In what follows, assume that $p = \frac{2}{3}$.

(c) [5] Suppose the mechanism is now an English auction. What is the probability that bidding eventually reaches H?

(d) [5] If you were a revenue-maximizing auctioneer, which mechanism (English or second-price sealed-bid action) would you choose? Justify your answer.

(e) [5] Suppose A, B, and C now agree to always bid L. What is B's expected profit if the auctioneer utilitizes a second-price sealed-bid auction? With what probability will B end up winning the object?