Intermediate Microeconomics
Econ 3101, Section 002
Homework 5-Solutions

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NOTE: Partial points are only to be awarded if the answers given are incorrect.

Question 1. ***Exchange.***
Consider an exchange economy with two goods and two consumers (indexed by i), each with endowments $e_i = (2, 2)$ for $i = 1, 2$.
(a) [10] Find all Pareto-optimal allocations when $u_1(x_1, x_2) = x_1 + 2x_2$ and $u_2(y_1, y_2) = 2y_1 + y_2$.

Ans: The set of Pareto-optimal allocations is given by $\{(0,0),(4,4)\} \cup \{(0,4),\{(x_2,y_2) \in R^2_+ : x_2 + y_2 = 4\}\} \cup \{(0,4),(4,0)\} \cup \{\{(x_1,y_1) \in R^2_+ : x_1 + y_1 = 4\},(4,0)\} \cup \{(4,4),(0,0)\}$.

(b) [15] Find all competitive equilibria when $u_1(x_1, x_2) = x_1 + 2x_2$ and $u_2(y_1, y_2) = 2y_1 + y_2$.

Ans: In equilibrium we have relative price $\frac{p_1}{p_2} = 1$ and allocations $\{(0,4),(4,0)\}$.

Question 2. ***Welfare.***
Suppose a cataclysmic event occurs and suddenly there are only 101 people left on the planet. You have been declared king (queen) and must now decide what to give to the rest of your (100) subjects. After taking your share, there are only 500 resource units left. Of these 100 people, 70 are old (X) while 30 are young (Y). For simplicity, assume all the old (young) people are treated the same, but that the old and young can be treated differently.
(a) [5] If you have a utilitarian welfare function $U(X, Y) = 70X + 30Y$, how much would the old people get as a group? What about the young?

Ans: The maximization problem is given by
max $U(X, Y) = 70X + 30Y$ s.t. $70X + 30Y = 500$.

This results in the following resource distribution: **Old(350)**, **Young(150)**.

(b) [5] Suppose you have a Rawlsian welfare function $U(X,Y) = min\{70X, 30Y\}$ instead. What would the old receive as a group? What about the young?

Ans: The maximization problem is given by
max $U(X,Y) = min\{70X, 30Y\}$ s.t. $70X + 30Y = 500$.

This results in the following resource distribution: **Old(250)**, **Young(250)**.

(c) [5] What if $U(X,Y) = (70X)^{0.2}(30Y)^{0.8}$? What do the old (young) get as a result?

Ans: The maximization problem is given by
max $U(X,Y) = (70X)^{0.2}(30Y)^{0.8}$ s.t. $70X + 30Y = 500$.

This results in the following resource distribution: **Old(100)**, **Young(400)**.

(d) [5] Take the concave utility function $U(X,Y) = (70X)^2 + (30Y)^2$. What amount gets allocated to the old? The young? (Hint: There may be more than one optimal outcome.)

Ans: The maximization problem is given by
max $U(X,Y) = (70X)^2 + (30Y)^2$ s.t. $70X + 30Y = 500$.

This results in either of the following resource distributions: **(Old,Young)=(500,0) or (0,500)**.

(e) [5] Finally consider the Nietzschean utility function $U(X,Y) = max\{70X, 30Y\}$. How much does each group receive in this case? (Hint: There may be more than one optimal outcome.)

Ans: The maximization problem is given by
max $U(X,Y) = max\{70X, 30Y\}$ s.t. $70X + 30Y = 500$.

This results in either of the following resource distributions: **(Old,Young)=(500,0) or (0,500)**.

Question 3. ***Asymmetric Information.***
(Varian) In Pot Hole, Georgia, 1000 people want to sell their used cars. These cars vary in quality. Original owners know exactly what their cars are worth. All used cars look the same to potential buyers until they have bought them; then they find out the truth. For any number X between 0 and 2000, the number of cares of quality lower than X is $\frac{X}{2}$. If a car is of quality

X, its original owner will be willing to sell it for any price greater than X. If a buyer knows that a car is of quality X, she will be willing to pay $X + 500$ for it. When buyers are not sure of the quality of a car, they are willing to pay its expected value, given their knowledge of the distribution of qualities on the market.

(a) [10] Suppose that everyone knows that all the used cars in Pot Hole are for sale. What would used cars sell for? Would every used car owner be willing to sell at this price? Which used cars would appear on the market?

Ans: Since it follows a uniform distribution between 0 and 2000, the expected value of used car quality is 1000. Knowing this, customers are willing to purchase a used car for 1500, so this ends up being the price of a used car. **Not every car owner** will be willing to sell at this price. **Only used cars with quality under** 1500 will be sold in this market.

(b) [10] Let X^* be some number between 0 and 2000 and suppose that all cars of quality lower than X^* are sold, but original owners keep all cars of qualit higher than X^*. What would buyers be willing to pay for a used car? At this price, which used cars would be for sale?

Ans: Since it follows a uniform distribution between 0 and X^*, the expected value of used car quality is $\frac{X^*}{2}$. Knowing this, customers are willing to purchase a used car for $\frac{X^*}{2} + 500$. **Only used cars with quality under** $\frac{X^*}{2} + 500$ will be sold in this market.

(c) [5] Write an equation for the equilibrium value of X^*, at which the price that the buyers are willing to pay is exactly enough to induce all cars of quality less than X^* to enter the market. Solve this equation for X^*.

Ans: From part (b), we know that all cars with quality less than $\frac{X^*}{2} + 500$ will enter the market. Hence the equation must be

$$\frac{X^*}{2} + 500 = X^* \tag{1}$$

Solving the equation for X^* yields $X^* = 1000$.

Question 4. *Auctions.*

Imagine a world where there are three bidders, A, B, and C, and their individual valuations of the object under consideration is H with probability p and L with probability $1 - p$, where $H > L$ and $0 < p < 1$. Ties are broken randomly. All proceeds go to the auctioneer.

(a) [5] If the mechanism is a second-price sealed-bid auction and $p = \frac{2}{3}$, what should A bid when he values the object at H? At L?

Ans: H**;** L**.**

(b) [5] Given the information in part (a), if the bidders don't collude, what is the probability that the auctioneer makes H? What will this probability be if $p = \frac{1}{2}$ instead?

Ans: The auctioneer makes H if **at least two bidders** bid H. This happens with probability $\frac{20}{27}$; for $p = \frac{1}{2}$, the probability is $\frac{1}{2}$.

In what follows, assume that $p = \frac{2}{3}$*.*
(c) [5] Suppose the mechanism is now an English auction. What is the probability that bidding eventually reaches H?

Ans: Bidding will eventually reach H only if there are **at least two bidders** who value the object at H. Thus the **probability is** $\frac{20}{27}$, the same as in part (b).

(d) [5] If you were a revenue-maximizing auctioneer, which mechanism (English or second-price sealed-bid action) would you choose? Justify your answer.

Ans: **Either mechanism** is fine as both yield the **same expected value or revenue** of $\frac{20}{27}H + \frac{7}{27}L$.

(e) [5] Suppose A, B, and C now agree to always bid L. With what probability will B end up winning the object, if no one cheats and the auctioneer utilitizes a second-price sealed-bid auction? What is B's expected profit?

Ans: Since all bidders bid L and ties are randomly broken, B wins with **probability** $\frac{1}{3}$. B's **expected profit is** $\frac{H-L}{3}$.