# Intermediate Microeconomics
# Econ 3101, Section 002
# Homework 4-Solutions

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**NOTE: Partial points are only to be awarded if the answers given are incorrect.**

**Question 1.** ***Monopoly.***
Consider a monopolist with the total cost function

$$TC(Q) = \frac{Q^2}{2} + 10Q + 20 \tag{1}$$

facing the market demand equation $Q = 70 - P$.
(a) [5] What is the profit-maximizing output and price for the monopolist? Calculate its profit.

Ans: The monopolist will solve $MR = MC$. Given that $MC = Q + 10$ and $MR = 70 - 2Q$, we have that $Q^* = 20$ and (using the demand equation) $P^* = 50$.

Profit is calculated using $\pi = TR - TC$ where $TR = P^*Q^* = 1000$ and $TC = \frac{Q^{*2}}{2} + 10Q^* + 20 = 420$. Hence $\pi = 580$.

(b) [5] What is the socially optimal output and price for this firm?

Ans: The socially optimal quantity is obtained by solving $D = MC$. From the information above we have $Q_e = 30$ and $P_e = 40$.

(c) [5] What is the deadweight loss generated by the monopolist?

Ans: To calculate DWL, first calculate $P' = MC(Q^*) = 30$ then use $DWL = \frac{1}{2}(P^* - P')(Q_e - Q^*)$ to get $DWL = 100$.

(d) [5] Suppose now that the monopolist is charged a profit tax of 25%. What is the profit-

maximizing output in this case? Compare this to what you obtained in part (a). Explain any similarities/differences.

Ans: Same as in part(a): $Q^* = 20$. This is because the tax is taken after profit; it does not affect the profit-maximizing decision.

(e) [5] Suppose the monopolist is charged a quantity tax instead. For what (nonzero) value(s) of the quantity tax will this tax system be equivalent to the profit tax system given in (d)? Here we define two tax systems to be equivalent if they result in the monopolist producing the same output. Justify your answer.

Ans: There is no quantity tax for which the two tax regimes will be equivalent. This is because quantity tax affects the profit-maximizing decision (while the profit tax does not).

(f) [5] Now suppose there are no taxes and the firm is a perfect price-discriminating monopolist. Calculate equilibrium output and quantity.

Ans: Same as in part (b): $Q_e = 30$ and $P_e = 40$.

(g) [5] Is there any deadweight loss in the case presented in part (f)? How is it different from the perfectly competitive outcome? (Hint: Think about who gains and loses as a result of these different market structures.)

Ans: No. In the case with perfect price discrimination, all the social surplus goes to the producer; this is split between consumer and producer in pure competition.

**Question 2.** ***Oligopoly.***
Let market demand be given by $P = 100 - Q$, and suppose the only two firms in the market have the following cost functions: $TC(y_1) = 20y_1$ and $TC(y_2) = 20y_2$.
(a) [5] Find the Cournot-Nash equilibrium.

Ans: The reaction functions are given by $y_1 = \frac{80-y_2}{2}$ and $y_2 = \frac{80-y_1}{2}$. In equilibrium we have $y_1 = \frac{80}{3}$, $y_2 = \frac{80}{3}$.

(b) [5] What is the market price and profits of the firms under Cournot-Nash equilibrium?

Ans: Price is determined by plugging the value derived in (a) into the market demand func-

tion $P = 100 - y_1 - y_2$. Hence $P = \frac{140}{3}$. Profits are calculated using $\pi = TR - TC$. $\pi_1 = \pi_2 = \frac{80^2}{9} \approx 711.11$.

(c) [5] Suppose now the two firms collude and form a cartel. Find the market price, total output, and joint profit of the cartel.

Ans: Total output is found by maximizing $(100 - Q)Q - 20Q$. $Q^* = 40$. Market price is taken from market demand: $P = 100 - Q$. $P^* = 60$. Profit is found using $\pi = TR - TC$. $\pi = 1600$.

(d) [5] Now suppose the first firm cheats while the second firm doesn't. What is the first firm's profit-maximizing output and how much profit does it make?

Ans: The cheating firm maximizes $(80 - y_1)y_1 - 20y_1$. Hence $y_1 = 30$. To calculate profit, the new price must first be calculated using $P = 100 - y_1 - y_2 = 50$. Then $\pi = TR - TC$ yields $\pi_1 = 50 * 30 - 20 * 30 = 900$.

(e) [5] What if the second firm cheats instead (while the first firm doesn't)? How much output will it produce? Calculate the resulting profit.

Ans: This is the same as what the first firm does in part (d) by symmetry.

(f) [5] Consider the scenario given in part (e). Firm 1 discovers firm 2 cheating and decides to following suit (so that both firms go down together). What will be the market price and profit made by each firm?

Ans: In following suit, firm 1 raises total output to 60 so that $P = 40$. By $\pi = TR - TC$, each firm only makes $\pi = 600$, which is necessarily lower than the collusive outcome.

(g) [5] Now suppose instead that the first firm sets its quantity first and the other firm follows. Find the quantities produced by each firm, the market price, and the firm's profits.

Ans: The reaction function for the follower is given by $y_2 = \frac{80-y_1}{2}$. The leader takes this into consideration so that he maximizes $(100 - y_1 - \frac{80-y_1}{2})y_1 - 20y_1$. This yields $y_1 = 40$. Plugging this back into the reaction function yields $y_2 = 20$. The market price is calculated using $P = 100 - y_1 - y_2$. $P = 40$. Profits are calculated using $\pi = TR - TC$. $\pi_1 = 800$, $\pi_2 = 400$.

**Question 3.** ***Game Theory.***

Consider the Colluder's Dilemma (an example of which is given in the previous question). Two firms that collude successfully to suppress output can charge a higher price and hence make a larger profit (than they would by competing against each other). There is, however, an incentive to cheat: by cheating, they make even more profit but only if the other firm doesn't retaliate. If it does, they both lose out. Suppose colluding brings both parties a profit of 1000. If only one firm cheats, it makes 1200 while its partner gets 600. If retaliation occurs (i.e. they both cheat), they each get 800.

(a) [10] Construct the payoff matrix for this game.

Ans:

| | Comply | Cheat |
|---|---|---|
| Comply | 1000, 1000 | 600, 1200 |
| Cheat | 1200, 600 | 800, 800 |

(b) [10] Find all pure-strategy Nash equilibrium (equilibria).

Ans:

| | Comply | Cheat |
|---|---|---|
| Comply | 1000, 1000 | 600, <u>1200</u> |
| Cheat | <u>1200</u>, 600 | <u>800</u>, <u>800</u> |

The only Nash equilbrium in this case is (Cheat, Cheat).

(c) [10] Find all purely-mixed Nash equilibrium (equilibria).

Ans: There are no purely-mixed Nash equilibria in this case. This is because there is a dominant strategy (Cheat) for both players.