Intermediate Microeconomics
Econ 3101, Section 002
Homework 3

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Question 1. ***Production Functions.***
(a) [10] What are the average products of labor and capital (AP_L, AP_K) for a firm with production function $f(L, K) = L^{0.6}K^{0.4}$? What about the marginal products (MP_L, MP_K)? Find the technical rate of substitution (TRS).
(b) [5] Does the production function in (a) exhibit decreasing, increasing, or constant returns to scale? Justify your answer.
(c) [5] Suppose a firm's production function is given by $f(L, K) = 3L + 5K$. If labor falls by three units, by how many units must the firm increase capital in order to produce the same amount of output?
(d) [5] Now suppose the firm's production function is $f(L, K) = max\{3L, 5K\}$ instead. If labor falls by three units, by how many units must capital increase in order to produce the same amount of output? Is there enough information to answer this question? Explain.

Question 2. ***Profit Maximization.***
This question deals with the problem of a profit-maximizing firm which produces a good with price p using two factors, L and K, that have prices w and r, respectively.
(a) [10] Let the firm's production function be given by $f(L, K) = L^2 + K^2$. Find the firm's supply function given that $p > r$ and $p > w$.
(b) [5] Suppose now that the firm's production function is $f(L, K) = min\{L, 2K\}$. If $r = 5$ and $w = 3$, how much output will the firm produce if $p = 2$? Justify your answer.
(c) [5] The technology used by the firm in part (b) has changed so that it now has the linear production function $f(L, K) = L + K$. If there is no change in the prevailing prices, what is its profit-maximizing output?
(d) [5] Suppose $r = w$, and the firm has the linear production function given in part (c). What must the output price p be (in terms of r and k) for the firm to want to produce as much output as possible? Explain.

Question 3. ***Cost Curves.***

(Guo) Suppose a firm uses the following technology to produce the good y:

$$y = f(L, K) = min\{\frac{L}{a_L}, \frac{K}{a_K}\} \tag{1}$$

The input prices are $w = 1$ and $r = 5$.

(a) [5] What do the parameters a_K and a_L mean?

(b) [10] Derive the firm's long-run total cost curve ($TC(y)$), the long-run average cost curve ($AC(y)$), and the long-run marginal cost curve ($MC(y)$).

(c) [10] Now suppose that in the short-run, capital is fixed at $K = 10$, and $a_K = 2$, $a_L = 4$ as before. Derive the firm's short-run total cost curve ($TC(y)$), the short-run average cost curve ($AC(y)$), and the short-run marginal cost curve ($MC(y)$).

Question 4. ***Industry Supply.***

Suppose we live in an alternate reality where the primary form of transportation is not the automobile but the gondola. The market for gondola services is purely competitive. Let the marginal cost of a gondola ride be constant at five dollars, and assume that each gondola can make 20 trips per day. The demand function for gondola rides is given by $D(p) = 1500 - 20p$, where demand is measured in rides per day, and price is measured in dollars.

(a) [5] What is the competitive equilibrium price per ride? What is the number of gondola rides per day in equilibrium? How many gondolas will there be in equilibrium?

(b) [5] An act of God (AoG) forces gondola production to grind to a halt. There are no alternative forms of transportation, so that the only way to get around is through the gondolas already in existence. Costs have not changed, but the act of God makes people afraid to be out on the streets, so that demand for gondolas becomes $D(p) = 2000 - 20p$. What is the equilibrium price post-AoG?

(c) [5] What will the profit per day, per gondola be in the post-AoG world?

(d) [5] Suppose the interest rate is 5% and that the costs, demand and the number of gondolas in part (b) go on indefinitely. Assuming there are no costs associated with becoming a gondola driver and that a gondola is intrinsically worthless, what will be the market price for gondolas? The buyer starts profiting from the gondola from the period he makes the purchase.

(e) [5] Riots have broken out because of the change in price, to the point where the government decides to dip into its reserves of gondolas to bring the price back to its original level (i.e. the price you found in part(a)). How many gondolas would the government have to provide?