Intermediate Microeconomics
Econ 3101, Section 003
Homework 3-Solutions

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NOTE: Partial points are only to be awarded if the answers given are incorrect.

Question 1. ***Monopoly.***
Consider a monopolist with the total cost function

$$TC(Q) = \frac{Q^2}{2} + 10Q + 20 \tag{1}$$

facing the market demand equation $Q = 70 - P$.
(a) [10] What is the profit-maximizing output and price for the monopolist? Calculate its profit.

Ans: The monopolist will solve $MR = MC$. Given that $MC = Q + 10$ and $MR = 70 - 2Q$, we have that $Q^* = 20$ and (using the demand equation) $P^* = 50$.

Profit is calculated using $\pi = TR - TC$ where $TR = P^*Q^* = 1000$ and $TC = \frac{Q^{*2}}{2} + 10Q^* + 20 = 420$. Hence $\pi = 580$.

(b) [5] What is the socially optimal output and price for this firm?

Ans: The socially optimal quantity is obtained by solving $D = MC$. From the information above we have $Q_e = 30$ and $P_e = 40$.

(c) [5] What is the deadweight loss generated by the monopolist?

Ans: To calculate DWL, first calculate $P' = MC(Q^*) = 30$ then use $DWL = \frac{1}{2}(P^* - P')(Q_e - Q^*)$ to get $DWL = 100$.

(d) [5] Now suppose there are no taxes and the firm is a perfect price-discriminating monopolist.

Calculate equilibrium quantity.

Ans: Same as in part (b): $Q = 30$.

(e) [10] Is there any deadweight loss in the case presented in part (f)? How is it different from the perfectly competitive outcome? (Hint: Think about who gains and loses as a result of these different market structures.)

Ans: No. In the case with perfect price discrimination, all the social surplus goes to the producer; this is split between consumer and producer in pure competition.

Question 2. *Oligopoly.*
Let market demand be given by $P = 100 - Q$, and suppose the only two firms in the market have the following cost functions: $TC(y_1) = 20y_1$ and $TC(y_2) = 20y_2$.
(a) [10] Find the Cournot-Nash equilibrium.

Ans: The reaction functions are given by $y_1 = \frac{80-y_2}{2}$ and $y_2 = \frac{80-y_1}{2}$. In equilibrium we have $y_1 = \frac{80}{3}$, $y_2 = \frac{80}{3}$.

(b) [5] What is the market price and profits of the firms under Cournot-Nash equilibrium?

Ans: Price is determined by plugging the value derived in (a) into the market demand function $P = 100 - y_1 - y_2$. Hence $P = \frac{140}{3}$. Profits are calculated using $\pi = TR - TC$. $\pi_1 = \pi_2 = \frac{80^2}{9} \approx 711.11$.

(c) [15] Suppose now the two firms collude and form a cartel. Find the market price, total output, and joint profit of the cartel.

Ans: Total output is found by maximizing $(100 - Q)Q - 20Q$. $Q^* = 40$. Market price is taken from market demand: $P = 100 - Q$. $P^* = 60$. Profit is found using $\pi = TR - TC$. $\pi = 1600$.

(d) [5] Now suppose the first firm cheats while the second firm doesn't. What is the first firm's profit-maximizing output and how much profit does it make?

Ans: The cheating firm maximizes $(80 - y_1)y_1 - 20y_1$. Hence $y_1 = 30$. To calculate profit, the new price must first be calculated using $P = 100 - y_1 - y_2 = 50$. Then $\pi = TR - TC$ yields

$\pi_1 = 50 * 30 - 20 * 30 = 900.$

(e) [5] What if the second firm cheats instead (while the first firm doesn't)? How much output will it produce? Calculate the resulting profit.

Ans: This is the same as what the first firm does in part (d) by symmetry.

(f) [10] Consider the scenario given in part (e). Firm 1 discovers firm 2 cheating and decides to following suit (so that both firms go down together). What will be the market price and profit made by each firm?

Ans: In following suit, firm 1 raises total output to 60 so that $P = 40$. By $\pi = TR - TC$, each firm only makes $\pi = 600$, which is necessarily lower than the collusive outcome.

(g) [15] Now suppose instead that the first firm sets its quantity first and the other firm follows. Find the quantities produced by each firm, the market price, and the firm's profits.

Ans: The reaction function for the follower is given by $y_2 = \frac{80-y_1}{2}$. The leader takes this into consideration so that he maximizes $(100 - y_1 - \frac{80-y_1}{2})y_1 - 20y_1$. This yields $y_1 = 40$. Plugging this back into the reaction function yields $y_2 = 20$. The market price is calculated using $P = 100 - y_1 - y_2$. $P = 40$. Profits are calculated using $\pi = TR - TC$. $\pi_1 = 800$, $\pi_2 = 400$.