Intermediate Microeconomics
Econ 3101, Section 002
Homework 3-Solutions

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Question 1. ***Production Functions.***

(a) [10] What are the average products of labor and capital (AP_L, AP_K) for a firm with production function $f(L,K) = L^{0.6}K^{0.4}$? What about the marginal products (MP_L, MP_K)? Find the technical rate of substitution (TRS).

Ans:

$$AP_K = \frac{f(L,K)}{K} = L^{0.6}K^{-0.6} \tag{1}$$

$$AP_L = \frac{f(L,K)}{L} = L^{-0.4}K^{0.4} \tag{2}$$

$$MP_K = \frac{\partial f}{\partial K} = 0.4L^{0.6}K^{-0.6} \tag{3}$$

$$MP_L = \frac{\partial f}{\partial L} = 0.6L^{-0.4}K^{0.4} \tag{4}$$

$$TRS = -\frac{MP_K}{MP_L} = -\frac{2L}{3K} \tag{5}$$

(b) [5] Does the production function in (a) exhibit decreasing, increasing, or constant returns to scale? Justify your answer.

Ans: It exhibits constant returns to scale. This can be seen from $f(aL, aK) = (aL)^{0.6}(aK)^{0.4} = aL^{0.6}K^{0.4} = af(L,K)$.

(c) [5] Suppose a firm's production function is given by $f(L,K) = 3L + 5K$. If labor falls by three units, by how many units must the firm increase capital in order to produce the same amount of output?

Ans: Capital must rise by $\frac{9}{5}$.

(d) [5] Now suppose the firm's production function is $f(L,K) = max\{3L, 5K\}$ instead. If labor falls by three units, by how many units must capital increase in order to produce the same amount of output? Is there enough information to answer this question? Explain.

Ans: There is not enough information to answer this question.

The following example is instructive:

- For $L = 3, K = 10, \Delta K = 0$ is necessary to maintain $f(L,K) = 50$ for $\Delta L = -3$
- For $L = 10, K = 3, \Delta K = 3$ is necessary to maintain $f(L,K) = 30$ for $\Delta L = -3$

Any example illustrating that ΔK is not constant is sufficient justification.

Question 2. ***Profit Maximization.***
This question deals with the problem of a profit-maximizing firm which produces a good with price p using two factors, L and K, that have prices w and r, respectively.
(a) [10] Let the firm's production function be given by $f(L,K) = L^2 + K^2$. Find the firm's supply function given that $p > r$ and $p > w$.

Ans: The supply function is not well-defined. **[7 pts]**. This is because the firm will want to produce an unlimited number of goods given that price is higher than input costs and the production function exhibits increasing returns to scale.

(Alternatively, a more formal justification would be: suppose the firm utilizes (K, L) to produces y. Then profit $\pi = py - rK - wL$. If the firm increases capital to K' so it produces $y' > y$, we must have $\pi' = py' - rK' - wL > \pi$ because $p(K'^2 - K^2) = p(K'-K)(K'+K) > p(K'-K) > r(K'-K)$ and rearranging after adding $pL^2 - wL$ to both sides yields the desired inequality $\pi' > \pi$).

(b) [5] Suppose now that the firm's production function is $f(L,K) = min\{L, 2K\}$. If $r = 5$ and $w = 3$, how much output will the firm produce if $p = 2$? Justify your answer.

Ans: The firm will not produce any output. Output price is below input costs and it will make negative profits (loss) if it produces a positive amount.

Alternatively, a mathematical argument would go like this:

- if $L \leq 2K, \pi = 2min\{L, 2K\} - 3L - 5K = 2L - 3L - 5K = -L - 5K < 0$
- if $L > 2K, \pi = 2min\{L, 2K\} - 3L - 5K = 4K - 3L - 5K = -3L - K < 0$

(c) [5] The technology used by the firm in part (b) has changed so that it now has the linear production function $f(L, K) = L + K$. If there is no change in the prevailing prices, what is its profit-maximizing output?

Ans: Same as above: profit maximizing output is zero. This follows from the following argument: suppose for contradiction that $L, K \geq 0$ and both are not zero at the same time so that the firm produces positive output. Then profit $\pi = 2(L + K) - 3L - 5K = -L - 3K < 0$ so the firm is better off producing nothing.

(d) [5] Suppose $r = w$, and the firm has the linear production function given in part (c). What must the output price p be (in terms of r and k) for the firm to want to produce as much output as possible? Explain.

Ans: If $p > r = w$, a firm with linear production function will want to produce as much output as possible as it makes more profit the more it produces (similar increasing returns to scale argument as in part(a)).

Question 3. ***Cost Curves.***
(Guo) Suppose a firm uses the following technology to produce the good y:

$$y = f(L, K) = min\{\frac{L}{a_L}, \frac{K}{a_K}\} \tag{6}$$

The input prices are $w = 1$ and $r = 5$.
(a) [5] What do the parameters a_K and a_L mean?

Ans: a_K is the amount of capital needed to produce one unit of output. Similarly, a_L is the amount of labor needed to produce one unit of output.

(b) [10] Derive the firm's long-run total cost curve ($TC(y)$), the long-run average cost curve ($AC(y)$), and the long-run marginal cost curve ($MC(y)$).

Ans: Since every unit of output requires a_L units of L and a_K units of K, the cost of every unit is $wa_L + ra_K$ and the cost of y units is

$$TC(y) = y(wa_L + ra_K) \tag{7}$$

$$AC(y) = \frac{TC(y)}{y} = wa_L + ra_K \tag{8}$$

$$MC(y) = TC'(y) = wa_L + ra_K \tag{9}$$

It is a general feature of constant returns to scale technology that average and marginal long-run cost curves are the same.

(c) [10] Now suppose that in the short-run, capital is fixed at $K = 10$, and $a_K = 2$, $a_L = 4$ as before. Derive the firm's short-run total cost curve ($TC(y)$), the short-run average cost curve ($AC(y)$), and the short-run marginal cost curve ($MC(y)$).

Ans: The firm cannot produce more than $\frac{K}{a_K} = \frac{10}{2} = 5$, and requires 4 units of labor for each unit of output. Hence the short-run cost curves for $0 \leq y \leq 5$ is given by

$$TC(y) = 5 \cdot 10 + 4 \cdot y = 50 + 4y \tag{10}$$

$$AC(y) = \frac{50 + 4y}{y} = \frac{50}{y} + 4 \tag{11}$$

$$MC(y) = TC'(y) = 4 \tag{12}$$

Question 4. ***Industry Supply.***

Suppose we live in an alternate reality where the primary form of transportation is not the automobile but the gondola. The market for gondola services is purely competitive. Let the marginal cost of a gondola ride be constant at five dollars, and assume that each gondola can make 20 trips per day. The demand function for gondola rides is given by $D(p) = 1500 - 20p$, where demand is measured in rides per day, and price is measured in dollars.

(a) [5] What is the competitive equilibrium price per ride? What is the number of gondola rides per day in equilibrium? How many gondolas will there be in equilibrium?

Ans: The price must equal marginal cost in equilibrium, so $P = 5$. From the demand equation, we get the number of gondola rides to be $D(5) = 1500 - 20 \cdot 5 = 1400$. As each gondola makes 20 trips per day, there will be $\frac{1400}{20} = 70$ gondolas in equilibrium.

(b) [5] An act of God (AoG) forces gondola production to grind to a halt. There are no alternative forms of transportation, so that the only way to get around is through the gondolas already in existence. Costs have not changed, but the act of God makes people afraid to be out on the streets, so that demand for gondolas becomes $D(p) = 2000-20p$. What is the equilibrium price post-AoG?

Ans: Since the number of gondolas stays the same, so will the number of gondola rides. Hence $1400 = 2000 - 20P$ from which we get $P = \frac{2000-1400}{20} = 30$.

(c) [5] What will the profit per day, per gondola be in the post-AoG world?

Ans: Each ride brings a profit of $30-5=25$ and each gondola makes 20 rides a day, hence daily profit per gondola is 500 dollars.

(d) [5] Suppose the interest rate is 5% and that the costs, demand and the number of gondolas in part (b) go on indefinitely. Assuming there are no costs associated with becoming a gondola driver and that a gondola is intrinsically worthless, what will be the market price for gondolas? The buyer starts profiting from the gondola from the period he makes the purchase.

Ans: The market price will be equal to the present value of all (present and future) profits that can be extracted from a gondola. This is equal to $\frac{500}{1-\frac{1}{1+0.05}} = \frac{500\cdot 1.05}{0.05} = 10500$.

(e) [5] Riots have broken out because of the change in price, to the point where the government decides to dip into its reserves of gondolas to bring the price back to its original level (i.e. the price you found in part(a)). How many gondolas would the government have to provide?

Ans: If the price is to go back to 5 dollars, there would have to be $\frac{2000-20(5)}{20} = 95$ gondolas in the market. This implies that the government would have to provide $95 - 70 = 25$ gondolas to effect the desired price change.