Intermediate Microeconomics
Econ 3101, Section 002
Homework 2-Solutions

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NOTE: Partial points are only to be awarded if the answers given are incorrect.

Question 1. ***Demand I.***
Consider the case where there are two goods, x and y, with prices p_x and p_y respectively. Suppose the consumer has income m.
(a) [10] Given that the consumer's utility function is given by $u(x,y) = min\{2x, 3y\}$, determine the consumer's demand for goods x and y.

Ans: The consumer's problem is given by
max $u(x,y) = min\{2x, 3y\}$ s.t. $p_x x + p_y y = m$.

Given this particular function, the consumer's optimal choice requires $y = \frac{2x}{3}$. Plugging this into the budget constraint yields the demand functions $x^* = \frac{3m}{3p_x+2p_y}$, $y^* = \frac{2m}{3p_x+2p_y}$.

(b) [10] Suppose now the consumer's utility function is $u(x,y) = 2x + 3y$ instead. Derive the consumer's demand for goods x and y.

Ans: The consumer's problem is given by
max $u(x,y) = 2x + 3y$ s.t. $p_x x + p_y y = m$, $x \geq 0$, $y \geq 0$.

Demand is given by:

- For $\frac{p_x}{p_y} > \frac{2}{3}$, $x^* = 0; y^* = \frac{m}{p_y}$
- For $\frac{p_x}{p_y} < \frac{2}{3}$, $x^* = \frac{m}{p_x}; y^* = 0$
- For $\frac{p_x}{p_y} = \frac{2}{3}, (x^*, y^*) = \{(x,y) : p_x x + p_y y = m, x \geq 0, y \geq 0\}$

(c) [5] What is the optimal consumption bundle for the consumer in part(b) if p_x=3, p_y = 5, and m=30?

Ans: $\frac{p_x}{p_y} = \frac{3}{5}$ Since $\frac{p_x}{p_y} \leq \frac{2}{3}$, $x^* = 10; y^* = 0$

Question 2. ***Demand II.***

(Adams) This question deals with the Stone-Geary utility function, $u(x_1, x_2) = (x_1 - \gamma_1)^{\beta_1}(x_2 - \gamma_2)^{\beta_2}$, an important generalization of the Cobb-Douglas utility function.

(a) [10] Consider the case when $\beta_1 = 1$, $\beta_2 = 1$, $\gamma_1 > 0$, and $\gamma_2 > 0$. Let the price of good x_1 be p_1 and the price of good x_2 be p_2. Assume the consumer has income m. Derive the consumer's demand for x_1.

Ans: The consumer's problem is given by:
max $u(x_1, x_2) = (x_1 - \gamma_1)^{\beta_1}(x_2 - \gamma_2)^{\beta_2}$ s.t. $p_x x + p_y y = m, x \geq 0, y \geq 0$

Assume that income is sufficiently large so that the solution is interior. Now $\frac{-\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = -\frac{p_1}{p_2}$. We also have $\frac{\partial u}{\partial x_1} = x_2 - \gamma_2$ $\frac{\partial u}{\partial x_2} = x_1 - \gamma_1$. From these three equations and the budget constraint we get $x_1 = \frac{m + \gamma_1 p_1 - \gamma_2 p_2}{2p_1}$

(b) [5] Using the information in part (a), can you determine whether goods x_1 and x_2 are gross substitutes, gross complements, or unrelated goods? Explain.

Ans: Gross complements. This is because $\frac{\partial x_1}{\partial p_2} = \frac{-\gamma_2}{2p_1} < 0$.

(c) [5] Suppose now that $\beta_1 = 1$, $\beta_2 = 1$, $\gamma_1 = 0$, and $\gamma_2 = 0$. What is the demand for x_1?

Ans: Plugging the values into the demand function obtained in (a), we get $x_1 = \frac{m}{2p_1}$.

(d) [5] Suppose the paramters take on the values given in part (c). If the utility function is $u(x_1, x_2) = lnx_1 + lnx_2$, will demand for x_1 be the same as that of part (c)? Why or why not?

Ans: It will be the same. This is because one utility function is simply an affine transformation of the other.

Question 3. ***Slutsky Equation.***

Consider a Robinson Crusoe (i.e. single agent) economy with demand function $d(p_x, p_y, m) = (\frac{-2p_x + 0.5m}{p_x}, \frac{2p_x + 0.5m}{p_y})$. The price of good x is denoted by p_x, that of good y by p_y, and income by m.

(a) [5] Is any of the two goods inferior? Justify your answer.

Ans: Both goods are normal. This is because $\frac{\partial d_x}{\partial m} = \frac{0.5}{p_x} > 0$ and $\frac{\partial d_y}{\partial m} = \frac{0.5}{p_y} > 0$.

(b) [5] Are there Giffen goods in this economy? Justify your answer.

Ans: There are no Giffen goods in this economy. This can be seen from the fact that $\frac{\partial d_x}{\partial p_x} = 0$ and $\frac{\partial d_y}{\partial p_y} = \frac{-0.5}{p_y^2} < 0$.

(c) [5] Let $p_1 = 5$, $m = 20$, and suppose p_2 decreases from 4 to 2. Find the change in the quantity demanded of good 2.

Ans: Calculating demand of good 1 for both prices yields $x_1(5, 4, 20) = 5$ and $x_1(5, 2, 20) = 10$. The change in quantity demanded is then $\Delta x_1 = 10 - 5 = 5$.

(d) [10] For the change in part (c), calculate the magnitude of the Slutsky substitution and income effects.

Ans: Find the income associated with the pivoted budget line: $\Delta m = d_y(5, 4, 20) \cdot \Delta p_y = 5 \cdot (-2) = -10$. Hence $m' = 20 - 10 = 10$.

Then compute $d_y(5, 2, 10) = \frac{2 \cdot 5 + 0.5 \cdot 10}{2} = 7.5$. Thus the substitution effect is $7.5 - 5 = 2.5$ and the income effect is $5 - (2.5) = 2.5$.

Question 4. ***Arbitrage.***

(Varian) In the textbook example discussing the market for oil, we assumed that there were zero production costs associated with getting oil out of the ground. Suppose instead that it costs five dollars per barrel to extract oil from the ground. Let the price in period t of one barrel of oil be p_t and let the interest rate be r, where $r > 0$.

(a) [5] What is the profit in extracting a barrel of oil in period t? What is its present value?

Ans: The profit would be $p_t - 5$ and its present value would be $\frac{p_t - 5}{(1+r)^t}$.

(b) [5] What is the present value of profits in period $t + 1$?

Ans: $\frac{p_{t+1} - 5}{(1+r)^{t+1}}$.

(c) [5] If the oil producing company is willing to supply oil in both periods t and $t+1$, what relation must hold? Express your answer in terms of a mathematical equation.

Ans: $\frac{p_t - 5}{(1+r)^t} = \frac{p_{t+1} - 5}{(1+r)^{t+1}}$.

(d) [10] Is the percentage rise in price between periods larger or smaller than the interest rate? Justify your answer.

Ans: Solving for p_{t+1} in terms of p_t using the relation obtained in part (c), we obtain $p_{t+1} = (1+r)p_t - 5r$. Since $r > 0$, the percentage rise in price between periods is smaller than the interest rate.