Intermediate Microeconomics
Econ 3101, Section 002
Homework 1-Solutions

Timothy Lim Uy

Question 1. ***Budget Constraint.***
Consider the case where there are two goods, x and y, with prices p_x and p_y respectively. Suppose the consumer has income m.
(a) [5] Write down an equation for the initial budget constraint.

Ans: $p_x x + p_y y = m$

(b) [5] Suppose now the price of the first good doubles, and the price of the second good becomes four times larger. Write down an equation for the new budget line in terms of original prices and income.

Ans: $(2p_x)x + (4p_y)y = m$

(c) [5] Now suppose the government decides to levy an ad valorem tax on the good x of 25% and provides a quantity subsidy of t on good y. Given original prices and income, write down the new budget line.

Ans: $(1.25p_x)x + (p_y - t)y = m$

(d) [5] Suppose $m = 150$, $p_x = 3$, and $p_x = 5$. The government decides to ration good x so no more than ten units can be consumed by a single individual. Give a mathematical formulation for the consumer's budget set.

Ans: $\{(x, y) : 3x + 5y \leq 150; x \leq 10\}$

Question 2. ***Preferences.***
(MWG) This question deals with the Condorcet Paradox, a central difficulty for the theory of decision making.
Consider a household consisting of Mom, Dad, and Junior. Suppose there are three choices for Friday night dinner: Chipotle, Campus Pizza, and Burger King. Mom says her ranking of the possibilities is (Chipotle, Campus Pizza, Burger King). Dad's preferences are given by (Campus Pizza, Burger King, Chipotle), while Junior would prefer Burger King first, then Chipotle, and finally Campus Pizza). They decide to take each pair of alternatives and let a majority vote determine the family rankings.
(a) [5] Dad suggests they first consider Chipotle vs. Burger King, then the winner of that contest vs. Campus Pizza. Which alternative will be chosen?

Ans: Burger King wins the first contest. Campus Pizza trumps Burger King and hence is chosen.

(b) [5] Mom suggests instead that they consider Campus Pizza vs. Burger King, and then the winner vs. Chipotle. Which gets chosen?

Ans: Campus Pizza wins the first contest. Chipotle trumps Burger King and hence is chosen.

(c) [5] What order should Junior suggest if he wants to get his favorite food for dinner?

Ans: He should suggest that they pit Chipotle and Campus Pizza first, with the winner to go up against Burger King. Chipotle wins the first contest, but Burger King eventually gets chosen.

(d) [5] Is the family's "collective preference", as determined by majority voting, transitive? Why or why not?

Ans: The family preference is given by:

- Burger King(B) $\gtrsim$ Chipotle(C)
- Chipotle(B) $\gtrsim$ Campus Pizza(A)
- Campus Pizza(A) $\gtrsim$ Burger King(B)

No, it is not transitive. If it were $B \gtrsim C, C \gtrsim A \Rightarrow B \gtrsim A$; but $A \gtrsim B$.

Question 3. ***Utility.***

Find the marginal rates of substitution between x and y for each of the following utility functions:

(a) [5] $u(x,y) = \alpha x + \beta y$

Ans: MRS is determined by:

$$MRS = -\frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} \tag{1}$$

Since $\frac{\partial U}{\partial x} = \alpha$, $\frac{\partial U}{\partial y} = \beta$ we have $MRS = -\frac{\alpha}{\beta}$.

(b) [5] $u(x,y) = (x-\alpha)^2 + (y-\beta)^2$

Ans: MRS is determined by:

$$MRS = -\frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} \tag{2}$$

Since $\frac{\partial U}{\partial x} = 2(x-\alpha)$, $\frac{\partial U}{\partial y} = 2(y-\beta)$ we have $MRS = -\frac{x-\alpha}{y-\beta}$.

(c) [5] $u(x,y) = ln(xy)$

Ans: MRS is determined by:

$$MRS = -\frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} \tag{3}$$

Since $\frac{\partial U}{\partial x} = \frac{1}{x}$, $\frac{\partial U}{\partial y} = \frac{1}{y}$ we have $MRS = -\frac{y}{x}$.

(d) [5] $u(x,y) = e^x + e^{2y}$

Ans: MRS is determined by:

$$MRS = -\frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} \tag{4}$$

Since $\frac{\partial U}{\partial x} = e^x$, $\frac{\partial U}{\partial y} = 2e^{2y}$ we have $MRS = -\frac{e^x}{2e^{2y}}$.

Question 4. ***Utility and Preferences I.***

Joe's utility function is given by $u_1(x,y) = x^2 + 2xy + y^2$. His buddy Al's utility function is given by $u_2(x,y) = x + y$.

(a) [5] Given the two bundles $(x,y) = (10,5)$ and $(x,y) = (15,10)$, which one will Joe choose? Explain.

Ans: The first bundle gives Joe utility $u_1(10,5) = 10^2 + 2(10)(5) + 5^2 = 225$ whereas the second bundle yields $u_1(15,10) = 15^2 + 2(15)(10) + 10^2 = 625$. Since $u_1(15,10) > u_1(10,5)$, Joe will choose $(15,10)$.

Note: An alternative way to justify Joe choosing (15,10) is to note that his utility function is monotonic (increasing) in both arguments (x and y) and that $(15,10) \gg (10,5)$.

(b) [5] Will Al choose the same bundle? Why or why not?

Ans: The first bundle gives Joe utility $u_2(10,5) = 10 + 5 = 15$ whereas the second bundle yields $u_2(15,10) = 15 + 10 = 25$. Since $u_2(15,10) > u_2(10,5)$, so Al will also choose $(15,10)$.

Note: An alternative way to justify Al choosing (15,10) is to note that his utility function is monotonic (increasing) in both arguments (x and y) and that $(15,10) \gg (10,5)$.

(c) [10] Do the two utility functions $u_1(x,y)$ and $u_2(x,y)$ represent the same preferences? That is, can you show that one utility function is a monotonic transformation of the other? Explain.

Ans: Yes. Joe's utility function is a monotonic transformation of Al's: $u_1(x,y) = f(u_2(x,y))$, where $f(z) = z^2$.

Question 5. ***Utility and Preferences II.***

(a) [5] What kind of preferences are represented by a utility function of the form $u(x,y) = 2\sqrt{x+y}$? Explain.

Ans: The utility function given represents perfect substitutes.

This can be seen by noting that $u(x,y) = f(x+y)$, where $f(z) = 2\sqrt{z}$, i.e. it is a monotonic transformation of the standard utility function representing perfect substitutes. Alternatively, one can also see that one unit of x gives the consumer just as much utility as one unit of y, so that they are perfectly substitutible.

(b) [5] Are the preferences represented by $u(x,y)$ in part (a) monotonic? Justify your answer.

Ans: Yes. This can be seen by taking the partial derivative of the utility function with respect to x and y and noting that it is positive or by showing that $u(x',y) > u(x,y)$ for $x' > x$

and $u(x, y') > u(x, y)$ for $y' > y$.

(c) [5] What kind of preferences are represented by a utility function of the form $v\,(x, y) = min\{2x, 2y\}$? Explain.

Ans: Perfect complements or Leontief preferences. A consumer with such a utility function would want to have an equal number of both goods, i.e. he wants to consume the two goods together.

(d) [5] Are the preferences represented by $v\,(x, y)$ in part (c) convex? Justify your answer.

Ans: Yes. This can be seen by noting that the upper contour sets are convex. (A mathematical argument using the fact that the utility function is concave is also acceptable.)