Diploma Growth Lecture 3

Dr. Timothy Uy
University of Cambridge

January 8, 2016

Recap of last lecture and transition to this one

- We discussed two models of growth with human capital accumulation and how they match up with data
- We showed the incorporating human capital into the Solow model improves its fit vis-a-vis the data
- Nevertheless, cross-country technological differences are required to generate differences in incomes across countries
- Thus far, we've assumed technological progress is exogenous; today we will consider the case with endogenous technological growth

Endogenous Growth: The Economics of Ideas

- Ideas are key to generating improvements in technology
- How ideas foster economic growth is through its nonrivalrous nature that allows for increasing returns
- Nonrivalrous goods are goods that can be used by many people simultaneously without loss; by contrast, use of rivalrous goods by one person precludes use of the same good by another person
- Rivalrous goods have to be produced each time they are sold; nonrivalrous goods only have to be produced once

Endogenous Growth: The Economics of Ideas

- Because only produced once, producing nonrivalrous goods require zero marginal cost and high fixed cost
- With high fixed cost and zero marginal cost, nonrivalrous good production then exhibits increasing returns to scale
- With constant returns to scale, costs are the same for each unit produced: this is true with constant marginal cost and no fixed cost
- With increasing returns to scale, costs go down with more units produced: this is true with zero marginal cost and high fixed cost

Endogenous Growth: The Economics of Ideas

- Given increasing returns to scale and high fixed costs, firms then have to earn positive profit to be able to pay the upfront fixed cost
- With perfect competition, firms make no profits and cannot pay the fixed cost upfront
- Under imperfect competition, firms can make positive profits as their products are differentiated and firm entry does not drive profits to zero

Endogenous Growth: The Romer Model

- Previous model were meant to explain the discrepancy in growth episodes across countries: rich and poor
- By contrast, the Romer model is designed to explain how advanced (rich) economies are able to sustain growth
- In the Romer model, technological progress is endogenously driven by research and development
- As in the Solow model, the two key components are the capital accumulation equation and the production function
- We will see however, that there is one important difference: the equation describing technological progress

Endogenous Growth: The Romer Model

- The production function takes the following form:
  $Y = K^{\alpha}(AL_Y)^{1-\alpha}$
- For a given level of A, the production function exhibits constant returns to scale (CRS) with respect to K and L
- Here, however, we will think of A as being another factor of production: in this case, the production function exhibits increasing returns
- To see this, note that doubling capital and labor doubles output (constant returns); however, doubling capital, labor, and the level of technology (i.e. stock of ideas) all at once more than doubles output (increasing returns)

Endogenous Growth: The Romer Model

- The capital accumulation equation is as in the Solow Model: $\dot{K} = s_K Y - dK$
- Similarly, labor grows exponentially at an exogenous rate n as before
- The big difference is in the growth of A: it is no longer exogenously set at g
- Instead, set $\dot{A} = \bar{\delta} L_A$, where $\bar{\delta}$ is the rate at which ideas are discovered and L_A is the amount of labor used to discover new ideas
- Because labor is now used for both production and innovation, there is an additional constraint: $L_A + L_Y = L$

Endogenous Growth: The Romer Model

- Let us parameterize $\bar{\delta} = \delta A^{\phi}$, so both labor and the stock of ideas itself determine the rate of technological progress

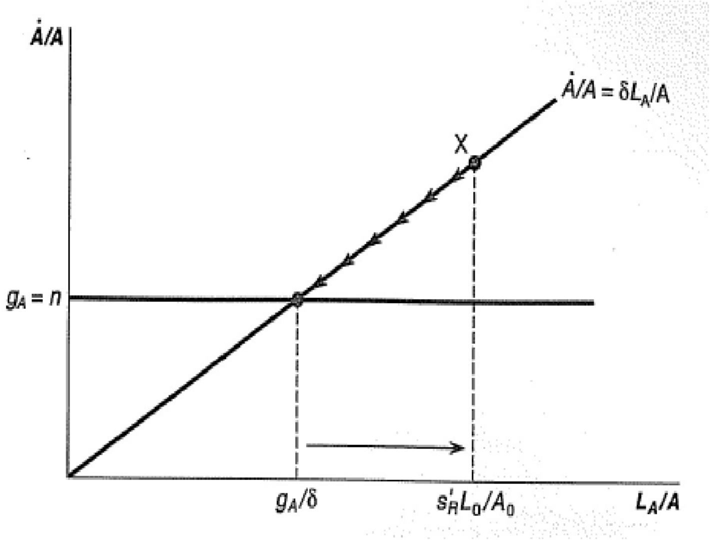
- We can now show that $g_y = g_k = g_A$, where $y = Y/L$ and $k = K/L$, i.e. the rate of technological progress is still the rate of growth for output and capital per worker

- Now to determine g_A: note that $\dot{A} = \delta L_A A^{\phi}$ hence $\frac{\dot{A}}{A} = \frac{\delta L_A}{A^{1-\phi}}$

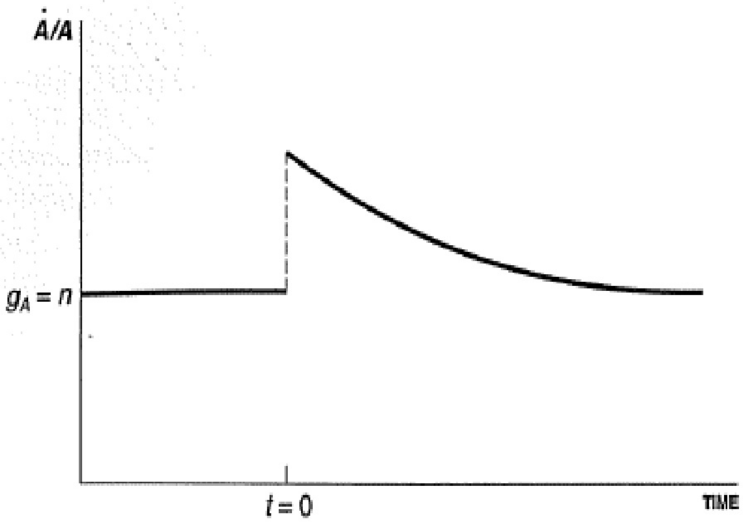
- We know that along a balanced growth path, $\dot{L}_A / L_A = \dot{L}/L = n$ as otherwise the number of researchers would exceed the total labor force

- For $\frac{\dot{A}}{A} = g_A$ to be constant, we need the numerator and denominator to grow at the same rate, hence after taking logs and derivatives, $\frac{\dot{L}_A}{L_A} = (1-\phi)\frac{\dot{A}}{A} \Rightarrow n = (1-\phi)g_A \Rightarrow g_A = \frac{n}{1-\phi}$

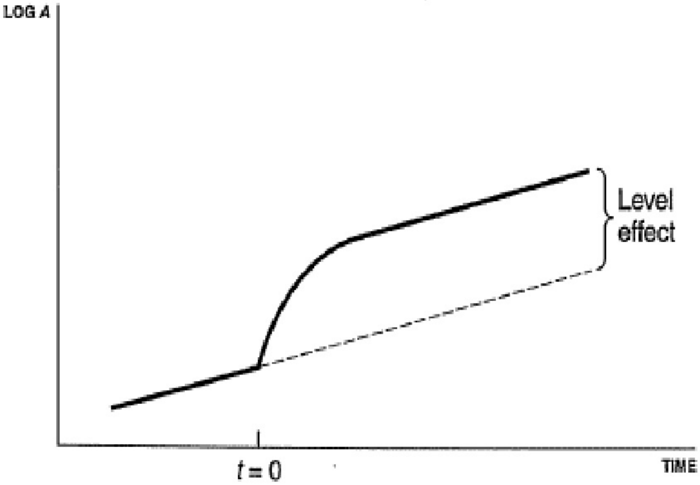
Growth vs Level Effects: Increase in R&D Share



Growth vs Level Effects: Increase in R&D Share



Growth vs Level Effects: Increase in R&D Share



Endogenous Growth: AK Models

- By noting that none of the parameters underlying $g_A = \frac{n}{1-\phi}$ can be affected by policies
- As in the Solow model, any effect that policy has is temporary and not permanent
- We will now attempt to construct a model where subsidies and taxes have long-run growth effects and not only level effects
- The AK model we will study will borrow heavily from the Jones (1995) paper

Endogenous Growth: The AK Model

- Consider the production function $Y = AK$, where A is now a constant
- The capital accumulation equation is as before $\dot{K} = sY - dK$
- In contrast to the previous models, the marginal product of capital never decreases in this case: it is always equal to A
- Dividing through the capital accumulation equation by K, we obtain
$$\frac{\dot{K}}{K} = s\frac{Y}{K} - d = sA - d$$

Endogenous Growth: The AK Model

- Given that A is now constant, Y grows at the same rate as K
- Hence we have

$$g_Y = \frac{\dot{Y}}{Y} = s\frac{Y}{K} - d = sA - d$$

- This is one of the key results of AK models: the growth rate of the economy is an increasing function of the investment rate s
- Hence government policies that increase the investment rate permanently increase the growth rate permanently as well

Endogenous Growth and Externalities

- To generate endogenous accumulation of knowledge, one needs increasing returns to scale in the production function
- One way to generate increasing returns is to introduce imperfect competition
- An alternative is to have externalities to production; this allows us to maintain perfect competition
- So even though capital and labor are paid their marginal product, the fact that the accumulation of knowledge is itself a by-product of production allows it be positive even when no output can be dedicated to it given perfect competition

Endogenous Growth and Externalities

- Take the standard production function $Y = BK^{\alpha}L^{1-\alpha}$
- If B is accumulated endogenously, production can still be characterized by increasing returns
- Suppose individual firms take the level of B as given; however, suppose in reality that the accumulation of capital generates new knowledge according to: $B = AK^{1-\alpha}$
- In this case, we have $Y = AKL^{1-\alpha}$ - increasing returns!

Conclusion

- We study two canonical models of endogenous growth: the Romer model and the AK model
- We show that the key to generating endogenous growth is generating increasing returns to scale in production
- We also discuss the two ways in which increasing returns to scale can be produced: imperfect competition and externalities