Diploma Growth Lecture 2

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Recap of last lecture and transition to this one

- We discussed exogenous and endogenous growth and the Kaldor facts in the last lecture
- We also talked about the Solow model and the properties of its steady state, comparative statics as well as dynamics
- In today's lecture we will talk about the Mankiw-Romer-Weil (MRW) model and see how it relates to material covered previously
- We will also see how the implications of the MRW model line up with some key empirical facts

The Mankiw-Romer-Weil Model

- In contrast to the Solow Model, suppose now that there is not only physical capital, but human capital as well
- Denoting human capital by H and its accumulation equation by $\dot{H} = s_H Y - dH$; physical capital accumulation follows $\dot{K} = s_K Y - dK$
- Production is typically characterized by a Cobb-Douglas production function

$$Y = K^{\alpha} H^{\beta} (AL)^{1-\alpha-\beta}$$

- Transform variables as before $\tilde{y} = \frac{Y}{AL}$, $\tilde{k} = \frac{K}{AL}$, $\tilde{h} = \frac{H}{AL}$
- We can then write the production function in terms of the transformed variables: $\tilde{y} = \tilde{k}^{\alpha} \tilde{h}^{\beta}$

The Mankiw-Romer-Weil Model

- We can write the capital accumulation equations in terms of transformed variables as before

$$\dot{\tilde{k}} = s_K \tilde{y} - (d + g + n)\,\tilde{k}$$
$$\dot{\tilde{h}} = s_H \tilde{y} - (d + g + n)\,\tilde{h}$$

- In steady state we have $\dot{\tilde{k}} = 0$, and $\dot{\tilde{h}} = 0$, yielding steady state capital in terms of output

$$\tilde{k}^* = \left(\frac{s_K}{d + g + n}\right)\tilde{y}^*$$
$$\tilde{h}^* = \left(\frac{s_H}{d + g + n}\right)\tilde{y}^*$$

The Mankiw-Romer-Weil Model

- Plugging these functions into the production function, we obtain a single equation for output per effective labor:

$$\tilde{y}^* = \tilde{k^*}^{\alpha}\tilde{h^*}^{\beta} = \left(\frac{s_K}{d+g+n}\right)^{\alpha}\tilde{y^*}^{\alpha}\left(\frac{s_H}{d+g+n}\right)^{\beta}\tilde{y^*}^{\beta}$$

- Grouping like terms we then obtain the final expression for output per effective worker

$$\tilde{y}^* = \tilde{k^*}^{\alpha}\tilde{h^*}^{\beta} = \left(\frac{s_K}{d+g+n}\right)^{\frac{\alpha}{1-\alpha-\beta}}\left(\frac{s_H}{d+g+n}\right)^{\frac{\beta}{1-\alpha-\beta}}$$

The Mankiw-Romer-Weil Model

- From the expression for output per effective worker we obtain

$$\tilde{k}^* = \left(\frac{s_K^{1-\beta} s_H^{\beta}}{d+g+n} \right)^{\frac{\alpha}{1-\alpha-\beta}}, \quad \tilde{h}^* = \left(\frac{s_K^{\alpha} s_H^{1-\alpha}}{d+g+n} \right)^{\frac{\beta}{1-\alpha-\beta}}$$

- Writing out the expression for output per worker, we obtain

$$y^* = A \left(\frac{s_K}{d+g+n} \right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{s_H}{d+g+n} \right)^{\frac{\beta}{1-\alpha-\beta}}$$

- We know that $A = A_0 g^t$. Substituting and taking logs yields

$$\log \frac{Y_t}{L_t} = \log A(0) + gt - \frac{\alpha+\beta}{1-\alpha-\beta} \log(d+g+n) + \frac{\alpha}{1-\alpha-\beta} \log(s_K) + \frac{\beta}{1-\alpha-\beta} \log(s_H)$$

The Mankiw-Romer-Weil Model

- Compare this with the expression for output per worker in the model with no human capital,

$$\log \frac{Y_t}{L_t} = \log A(0) + gt - \frac{\alpha}{1-\alpha}\log(d+g+n) + \frac{\alpha}{1-\alpha}\log(s_K)$$

- First, note that the signs of the coefficients in both models are the same: negative for population growth and positive for savings

- Second, given $\alpha = 1/3$ and $\beta = 1/3$, we have that the magnitude of the elasticities in the original model with no human capital are $\frac{\alpha}{1-\alpha} = 1/2$, while the elasticities in the augmented model are $\frac{\beta}{1-\alpha-\beta} = \frac{\alpha}{1-\alpha-\beta} = 1$

OLS Regression: Solow

TABLE I
ESTIMATION OF THE TEXTBOOK SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	5.48	5.36	7.97
	(1.59)	(1.55)	(2.48)
ln(I/GDP)	1.42	1.31	0.50
	(0.14)	(0.17)	(0.43)
$\ln(n + g + \delta)$	−1.97	−2.01	−0.76
	(0.56)	(0.53)	(0.84)
$\overline{R}^2$	0.59	0.59	0.01
s.e.e.	0.69	0.61	0.38
Restricted regression:			
CONSTANT	6.87	7.10	8.62
	(0.12)	(0.15)	(0.53)
$\ln(I/GDP) - \ln(n + g + \delta)$	1.48	1.43	0.56
	(0.12)	(0.14)	(0.36)
$\overline{R}^2$	0.59	0.59	0.06
s.e.e.	0.69	0.61	0.37
Test of restriction:			
p-value	0.38	0.26	0.79
Implied α	0.60	0.59	0.36
	(0.02)	(0.02)	(0.15)

Note. Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05.

OLS Regression: MRW

TABLE II
ESTIMATION OF THE AUGMENTED SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	6.89	7.81	8.63
	(1.17)	(1.19)	(2.19)
ln(I/GDP)	0.69	0.70	0.28
	(0.13)	(0.15)	(0.39)
$\ln(n+g+\delta)$	−1.73	−1.50	−1.07
	(0.41)	(0.40)	(0.75)
ln(SCHOOL)	0.66	0.73	0.76
	(0.07)	(0.10)	(0.29)
$\overline{R}^2$	0.78	0.77	0.24
s.e.e.	0.51	0.45	0.33
Restricted regression:			
CONSTANT	7.86	7.97	8.71
	(0.14)	(0.15)	(0.47)
$\ln(I/GDP) - \ln(n+g+\delta)$	0.73	0.71	0.29
	(0.12)	(0.14)	(0.33)
$\ln(SCHOOL) - \ln(n+g+\delta)$	0.67	0.74	0.76
	(0.07)	(0.09)	(0.28)
$\overline{R}^2$	0.78	0.77	0.28
s.e.e.	0.51	0.45	0.32
Test of restriction:			
p-value	0.41	0.89	0.97
Implied α	0.31	0.29	0.14
	(0.04)	(0.05)	(0.15)
Implied β	0.28	0.30	0.37
	(0.03)	(0.04)	(0.12)

Note. Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960–1985. $(g+\delta)$ is assumed to be 0.05. SCHOOL is the average percentage of the working-age population in secondary school for the period 1960–1985.

Another Model with Human Capital

- In contrast to the previous model where human capital is measured in output terms (as it be accumulated like physical capital), consider a model where human capital is measured in units of time (number of years)

- Specifically, suppose the production function is given by $Y = K^{\alpha}(AH)^{1-\alpha}$

- As before A grows exogenously at rate g; similarly, physical capital accumulation follows $\dot{K} = s_K Y - dK$

- Let L denote the total amount of unskilled labor in the economy;Unskilled labor learning skills for time u generates skilled labor H according to

$$H = e^{\psi u} L$$

Another Model with Human Capital

- Normalize the variables as follows: $y = Y/L$, $k = K/L$, and $h = H/L$.
- We obtain the normalized production function $y = k^{\alpha}(Ah)^{1-\alpha}$
- We also have the accumulation equations $h = e^{\psi u}$ (a constant) and $\dot{k} = s_K y - (d + n)k$
- Now normalize the variables so as to be able to solve the balanced growth path: $\tilde{y} = y/Ah$, $\tilde{k} = k/Ah$, etc

Another Model with Human Capital

- We can write as before: $\tilde{y} = \tilde{k}^{\alpha}$ and $\dot{\tilde{k}} = s_K \tilde{y} - (d + g + n)\,\tilde{k}$

- Given that the normalized physical accumulation equation is the same as the Solow Model, we know that in steady state

$$\tilde{k}^* = \left(\frac{s_K}{d+g+n}\right)^{\frac{1}{1-\alpha}}, \quad \tilde{y}^* = \left(\frac{s_K}{d+g+n}\right)^{\frac{\alpha}{1-\alpha}}$$

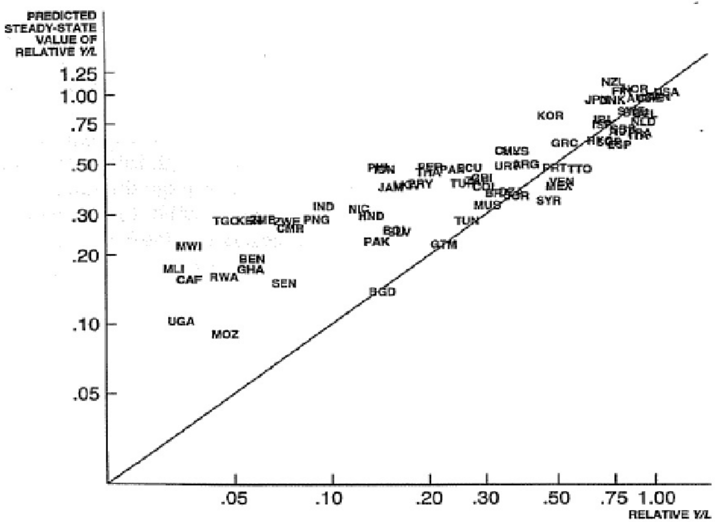
- The main difference between this model and Solow can be seen in the process for output per worker

$$y^* = \left(\frac{s_K}{d+g+n}\right)^{\frac{\alpha}{1-\alpha}} hA$$

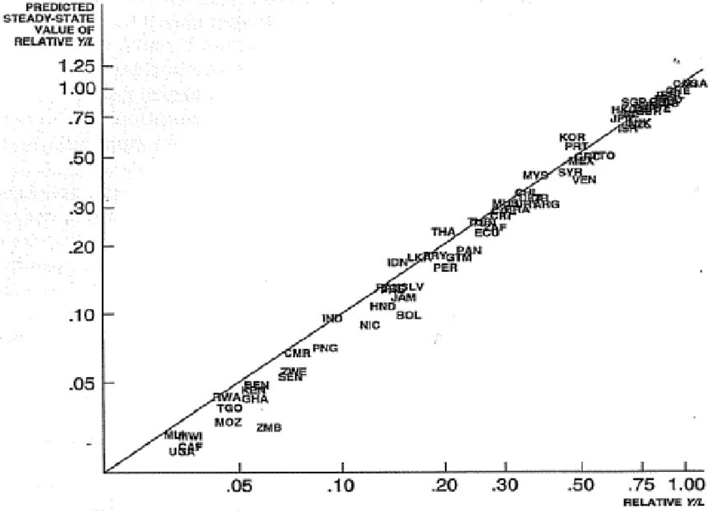
Another Model with Human Capital

- Given this process for output per worker, we can then see that this model predicts that countries can also be rich or poor depending on the amount of time they spend acquiring skills h
- If h is high as more time is spent acquiring skills u or because skilled labor is more efficiently acquired ψ, countries are richer
- As before, countries that save/investment more s_K, have lower population growth n, and have better technology A are richer
- This suggests that if savings/investment and population growth only partially account for the gap between rich and poor countries' incomes, then human capital is an alternative explanation that can help bridge the gap

Matching Model with Facts: Same Technology



Matching Model with Facts: Different Technology



Matching Model with Facts

TABLE 3.1 DATA AND PREDICTIONS FOR THE NEOCLASSICAL MODEL

	y/y_{US} actual 1990	y/y_{US} predicted SS value	s_K	u	n	$\hat{A}_{90}$
U.S.A.	1.00	1.00	0.210	11.8	0.009	1.00
West Germany	0.80	0.83	0.245	8.5	0.003	1.02
Japan	0.61	0.71	0.338	8.5	0.006	0.76
France	0.82	0.85	0.252	6.5	0.005	1.28
U.K.	0.73	0.76	0.171	8.7	0.002	1.10
Argentina	0.36	0.30	0.146	6.7	0.014	0.61
India	0.09	0.10	0.144	3.0	0.021	0.30
Zimbabwe	0.07	0.06	0.131	2.6	0.034	0.20
Uganda	0.03	0.02	0.018	1.9	0.024	0.25
Hong Kong	0.62	0.77	0.195	7.5	0.012	1.25
Taiwan	0.50	0.64	0.237	7.0	0.013	0.99
South Korea	0.43	0.59	0.299	7.8	0.012	0.74

SOURCE: Penn World Tables Mark 5.6, an update of Summers and Heston (1991), and author's calculations.

Note: The investment rates and population growth rates are averages for the period 1980–90. u denotes the average years of schooling of the labor force in 1985. $\hat{A}_{90}$ reports the estimated ratio of A/A_{US} in 1990. The second column of data reports predicted steady-state relative income using this data, as discussed in the text.

Conclusion

- We develop two models of exogenous growth with human capital accumulation
- We show how the model with human capital captures the key dimensions of the data better than the model without human capital
- Nevertheless, we find that sizeable differences in technology across countries are required to explain why some countries are significantly richer than others