Diploma Growth Lecture 1

Dr. Timothy Uy
University of Cambridge

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What is this course about?

- This course is about economic growth
- Economic growth occurs when people take resources and rearrange them in ways that are more valuable - Paul Romer
- We are going to be discussing two main types of economic growth: exogenous and endogenous growth

Exogenous vs Endogenous Growth

- Modern models of economic growth have at least two types of agents: cosnumers and producers
- The mechanism through which growth occurs relies on more effective production
- Production is typically characterized by a Cobb-Douglas production function

$$f(K, L) = AK^{\alpha}L^{1-\alpha}$$

- Exogenous vs endogenous growth is about the process for A: in the endogenous case, A grows endogenously

Kaldor Facts

- KF1: Output per worker grows continuously, with no secular tendency for rate of productivity growth to decline
- KF2: The capital-labor ratio shows continuous growth
- KF3: The rate of return on capital is stable
- KF4: The capital-output ratio is stable
- KF5: The shares of labor and capital in GDP remain stable
- KF6: There is significant variation in rate of productivity growth across countries

Matching Facts to Models

- Consider the production function for output at time t, Y_t:

$$Y_t = f(K_t, L_t) = A_t K_t^{\alpha} L_t^{1-\alpha}$$

- It relies on three inputs: technology/productivity A_t, capital K_t, and labor L_t, and parameter α

- Given this model specification of the production function, Kaldor Fact 1 (KF1) can be translated as $\frac{\dot{Y}}{L} / \frac{\dot{Y}}{L} = \kappa_1 > 0$ where $\dot{a}/a$ denotes growth rate of a

- Similarly, KF2 can be written as $\frac{\dot{K}}{L} / \frac{\dot{K}}{L} = \kappa_2 > 0$

Matching Facts to Models

- Proceeding in this way, we can write

$$\text{KF4: } \frac{\dot{K}}{Y} / \frac{K}{Y} = 0$$

$$\text{KF5: } \frac{rK}{Y} = \alpha, \ \frac{wL}{Y} = 1 - \alpha, \quad \alpha \text{ constant}$$

- Interpretation 1: we can see that the Cobb Douglas model specification is not a bad approximation given the constant factor shares (KF5)

- Interpretation 2: we can also see given KF1, KF2, and KF4 that population or labor force growth is potentially an important driver of growth

Model 1: Solow Model

- The production function is just one component of a full-blown general equilibrium model
- General equilibrium models of economic growth typically have firm and household optimization problems and market clearing
- One then has to solve for both the set of allocations (quantities) and prices
- First we will start with a classic model of economic growth: the Solow Model
- This is a useful benchmark as it allows us to think about issues on growth using a very simple framework
- We will see, however, that it abstracts from certain decisions that may be important for understanding economic policies that can potentially be used to stimulate growth

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Model 1: Solow Model

- Production function has labor-augmenting technology: $Y = K^{\alpha}(AL)^{1-\alpha}$

- Capital depreciates at rate d and consumers save fraction s of their income

- We can then write the capital accumulation equation as

$$\dot{K} = sY - dK$$

- Labor or population growth rate is n, and technology growth rate is exogenously set to g, i.e.

$$\frac{\dot{L}}{L} = n, \quad \frac{\dot{A}}{A} = g$$

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Model 1: Solow Model

- It is useful to define the following transformed variables $\tilde{k} = K/AL$ and $\tilde{y} = Y/AL$

- Let us rewrite the capital accumulation equation in terms of transformed variables

$$\frac{\dot{\tilde{k}}}{\tilde{k}} = s\tilde{k}^{\alpha-1} - (d + g + n), \tag{4}$$

- To obtain this equation, first note that we can write the original capital accumulation equation as

$$\frac{\dot{K}}{K} = sY/K - d = s\tilde{y}/\tilde{k} - d$$

Model 1: Solow Model

- Further, given that $Y = K^\alpha (AL)^{1-\alpha}$ we have $\frac{Y}{AL} = \left(\frac{K}{AL}\right)^\alpha$, i.e. $\tilde{y} = \tilde{k}^\alpha \Rightarrow \tilde{y}/\tilde{k} = \tilde{k}^{1-\alpha}$

$$\frac{\dot{K}}{K} = sY/K - d = s\tilde{y}/\tilde{k} - d = s\tilde{k}^{1-\alpha} - d$$

- Now it suffices to write the left hand side in terms of k. To do this, note that

$$\log \tilde{k} = \log K - \log A - \log L \Rightarrow \frac{\dot{\tilde{k}}}{\tilde{k}} = \frac{\dot{K}}{K} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L}$$

- Hence we have, as desired

$$\frac{\dot{\tilde{k}}}{\tilde{k}} = \frac{\dot{K}}{K} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L} = s\tilde{k}^{1-\alpha} - d - g - n$$

Characterizing Solow Model

- A sample model implication: $\frac{\dot{y}}{y} = g + \alpha \frac{\dot{\tilde{k}}}{\tilde{k}}$

- To show this, first note that because $\tilde{y} = \tilde{k}^{\alpha}$, we have

$$\log y = \alpha \log \tilde{k} \Rightarrow \frac{\dot{\tilde{y}}}{\tilde{y}} = \alpha \frac{\dot{\tilde{k}}}{\tilde{k}}$$

- Then to complete the argument, note that $y = Y/L = A\tilde{y}$ so we obtain as desired

$$\log y = \log A + \log \tilde{y} \Rightarrow \frac{\dot{y}}{y} = g + \frac{\dot{\tilde{y}}}{\tilde{y}} = g + \alpha \frac{\dot{\tilde{k}}}{\tilde{k}}$$

Characterizing Solow Model: Steady State

- The steady state can be characterized by setting $\frac{\dot{\tilde{k}}}{\tilde{k}} = 0$
- This yields $s\left(\tilde{k}^*\right)^{\alpha-1} = (d+g+n)$ hence in steady state,

$$\tilde{k}^* = \left(\frac{s}{d+g+n}\right)^{\frac{1}{1-\alpha}}$$

- We can also characterize steady-state output per effective worker

$$\tilde{y^*} = \tilde{k^*}^{\alpha} = \left(\frac{s}{d+g+n}\right)^{\frac{\alpha}{1-\alpha}}$$

Characterizing Solow Model: Comparative Statics

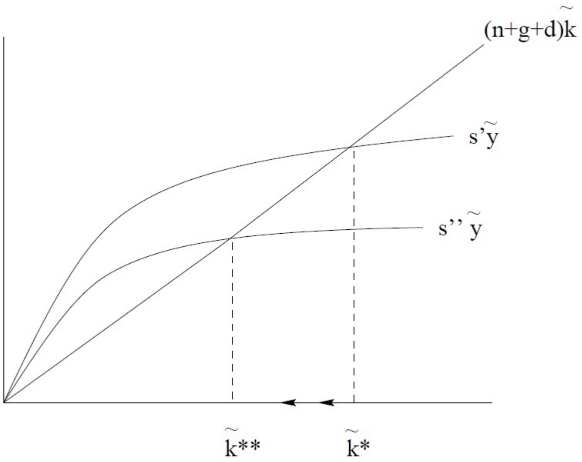
- It is easy to show that $\frac{d\tilde{k}^*}{ds} > 0$ and $\frac{d\tilde{y}^*}{ds} > 0$ so a higher saving/investment rate increases steady state capital and output per effective worker

- Similarly, lower population growth increases the capital and output per effective worker: $\frac{d\tilde{k}^*}{dn} < 0$ and $\frac{d\tilde{y}^*}{dn} < 0$

- These results seem to justify the argument that countries that want to grow should implement policies that stimulate investment and reduce the rate of population growth

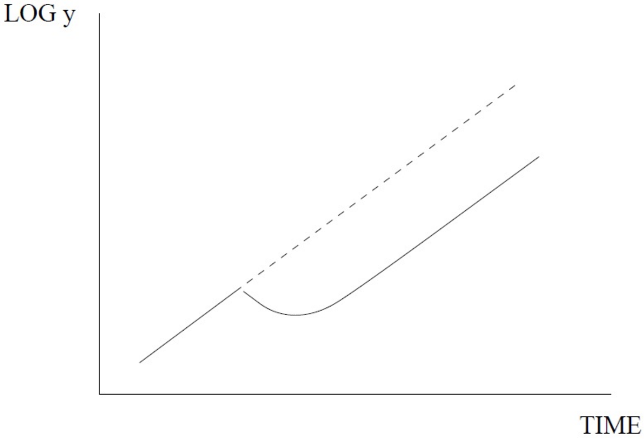
Characterizing Solow Model: Dynamics after change in s

Suppose we are now interested in the effects of a lower saving rate on the dynamics of capital per effective worker



Characterizing Solow Model: Dynamics after change in s

Lower s generates the following dynamics in output



Characterizing Solow Model: Dynamics after change in s

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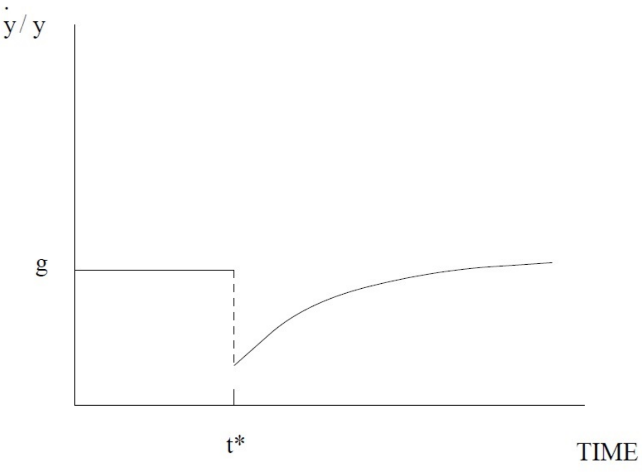

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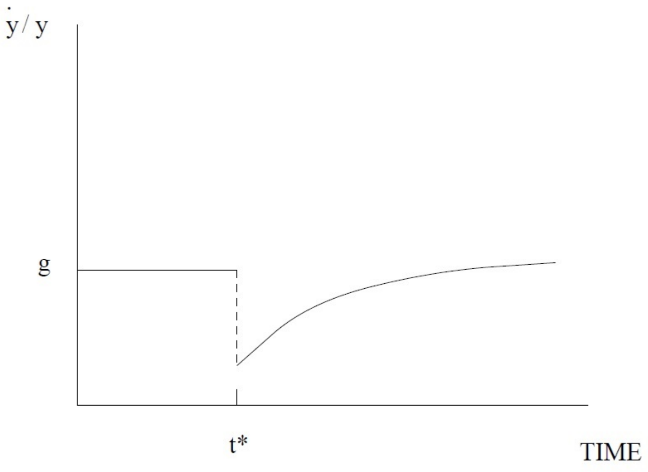
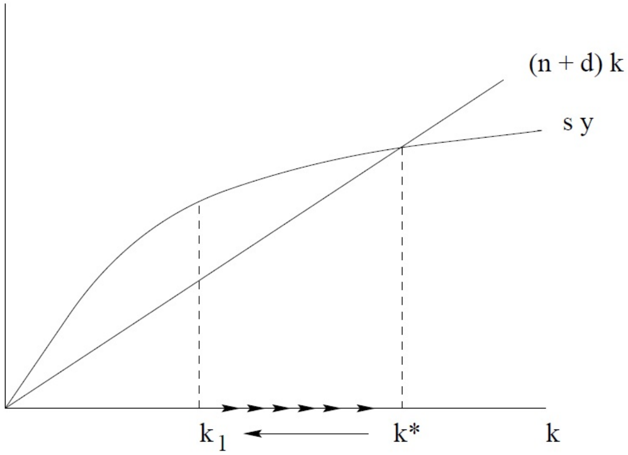


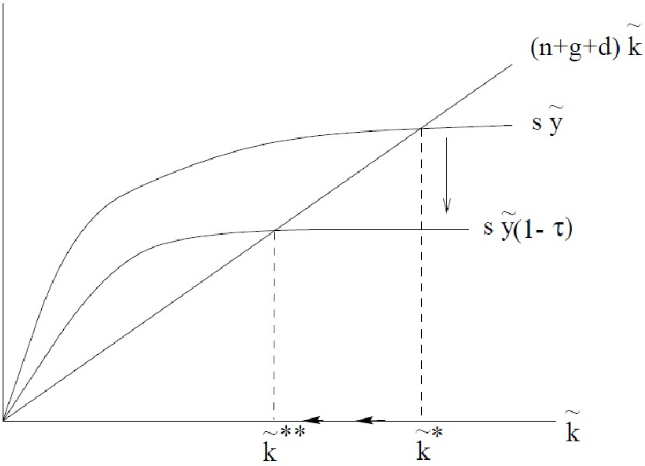
Characterizing Solow Model: Dynamics after change in L

Suppose we are now interested in the effects of a higher labor force on the dynamics of capital per effective worker



Characterizing Solow Model: Dynamics after income tax

Suppose we are now interested in the effects of an income tax on the dynamics of capital per effective worker



Key Concepts and Conclusions

In this lecture, we discussed the following concepts

- Exogenous and endogenous models of economic growth
- Kaldor facts and linking facts to models
- Solow model steady state, comparative statics, and dynamics after changes in savings rate, labor force, and income tax

Make sure you understand how they fit together so as to be able to build on them in the next lecture